Neural Quantum States for the Interacting Hofstadter Model with Higher Occupations and Long-Range Interactions

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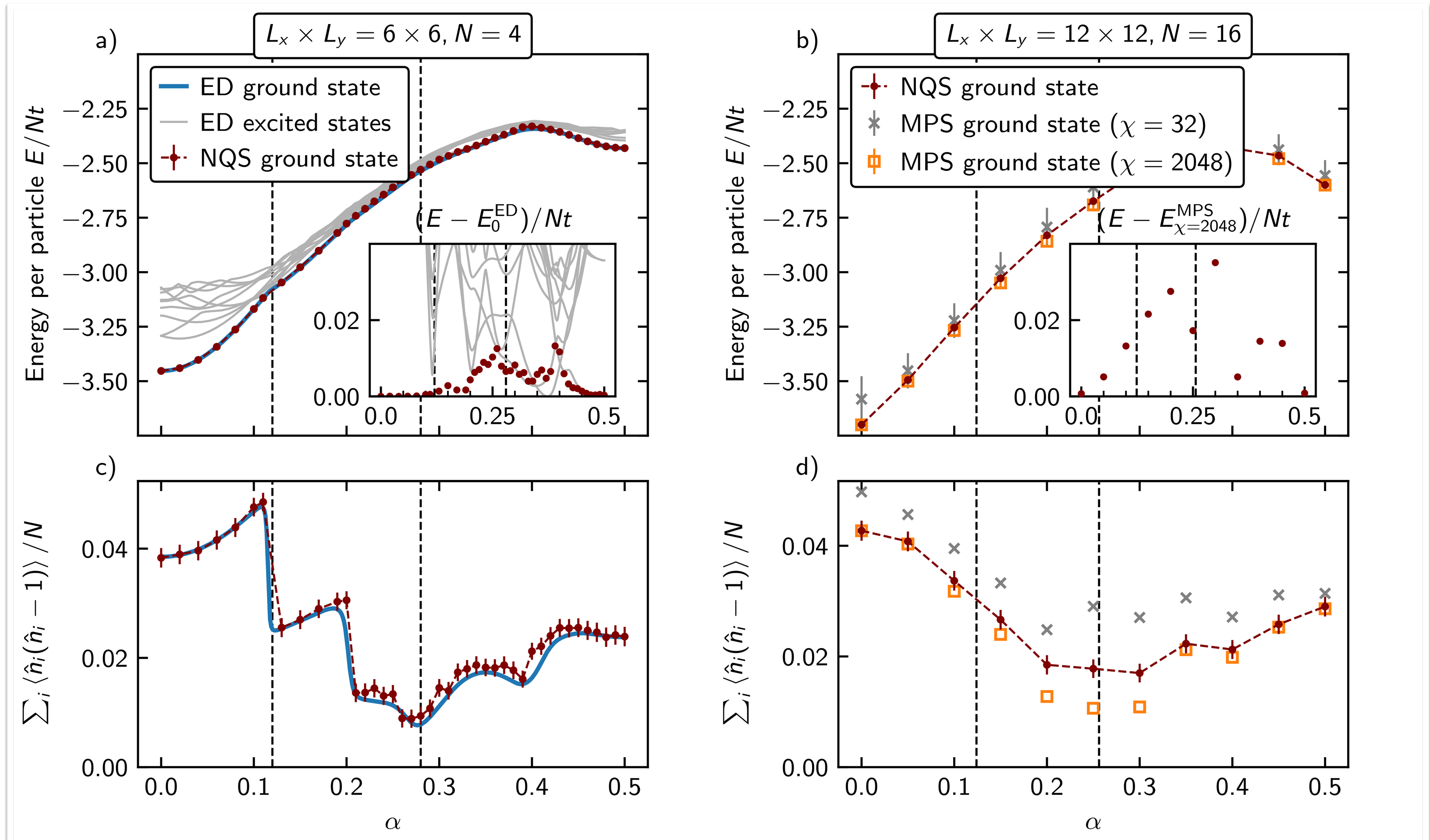
Introduction

Neural quantum states (NQS) have gained significant interest due to their representational power and potential to rival state-of-the-art numerical techniques, especially for large, two-dimensional quantum systems [1]. This study uses recurrent neural network (RNN) wave functions to model quantum many-body systems [2]. Specifically, we employ a 2D tensorized gated RNN to explore the bosonic Hofstadter model with a variable Hilbert space cut-off and long-range interactions. Benchmarking the RNN-NQS for the Hofstadter-Bose-Hubbard (HBH) Hamiltonian, we find that it efficiently captures most ground state properties. We further analyze a more challenging Hofstadter model with long-range interactions, which describes Rydberg-dressed atoms subject to a synthetic magnetic field. Studying systems up to 12×12 sites, we identify a bubble and Wigner crystal phase in addition to the HBH-regime. Here, the bubble crystal phase offers a starting point for the search of clustered liquid phases with potentially non-Abelian anyon excitations. This work demonstrates that NQS is an efficient, reliable tool for simulating complex quantum systems with the ability to simulate long-range interactions.

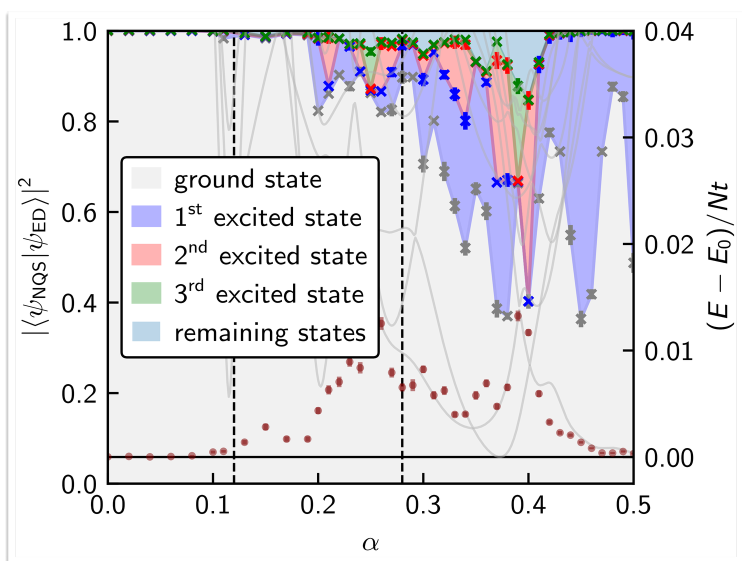
Hofstadter-Bose-Hubbard Model

To benchmark the NQS we apply the algorithm to the paradigmatic Hofstadter model with on-site interactions [4]:

$$\hat{H} = -t \sum_{x,y} (\hat{a}^\dagger_{x+1,y}\hat{a}_{x,y} + h.c.) - t \sum_{x,y} (\hat{a}^\dagger_{x,y+1}\hat{a}_{x,y} e^{i2\pi\alpha x} + h.c.) + \frac{U}{2}\sum_{x,y} \hat{n}_{x,y}(\hat{n}_{x,y} - 1).$$



The RNN-NQS obtains reliable estimations for observables in the smaller system. For the larger system, its results closely match the benchmark method, except for α values close to the topologically ordered Laughlin state ($\nu_{\frac{1}{2}} = \frac{N}{2N-1} = \frac{N}{\alpha N_{\text{plaquette}}}$). Note that MPS ($\chi = 32$) has a comparable number of variational parameters to the NQS, while performing worse.



The overlap with exact states offers insights into the trained RNN-NQS. At lower α values, the NQS achieves a perfect ground state overlap, but as α increases, contributions from other low-energy states grow.

Conclusion

RNN-NQS in FQH Systems: We successfully simulated fractional quantum Hall systems with RNN-NQS, which accurately captured most ground state properties on both 6×6 and 12×12 lattices.

Phase Diagram Findings: In addition to the Hofstadter-Bose-Hubbard regime, we identified a Wigner crystal and bubble crystal phase in the Hofstadter model with long-range interactions.

NQS Benefits: Efficient for systems with long-range interactions and high local occupations, complementing traditional methods specialized for short-range interactions.

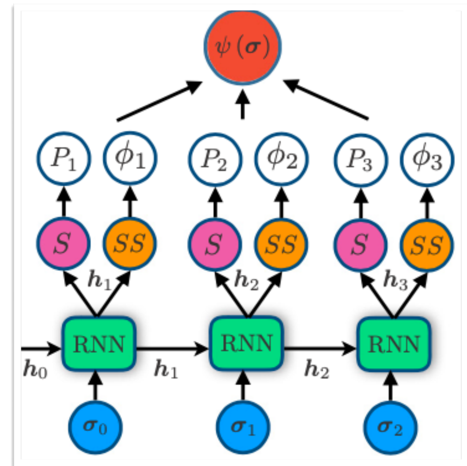
NQS-Model

We use a 2D tensorized gated RNN structure that allows to sample directly from the wave function [2][3]. The wave function is defined as follows:

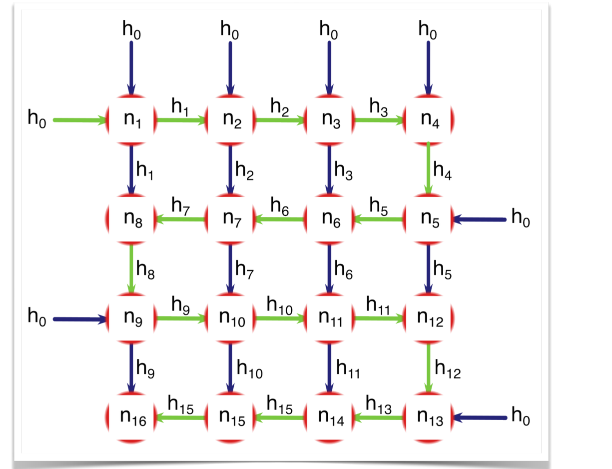
$$|\Psi\rangle = \sum_n \psi_\lambda(n)\,|n\rangle = \sum_n e^{i\phi_\lambda(n)}\sqrt{P_\lambda(n)}\,|n\rangle,$$

with $P_\lambda(n) = \prod_i p_\lambda(n_i\,|\,n_{j<i})$ and $\Phi_\lambda(n) = \sum_i \phi_\lambda(n_i\,|\,n_{j<i})$.

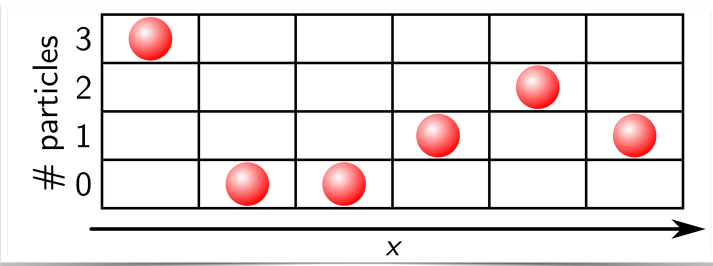
To obtain the conditional probability $p_\lambda(n_i\,|\,n_{j<i})$ and the phase $\phi_\lambda(n_i\,|\,n_{j<i})$, the local hidden state of the RNN is interpreted by linear layers with softsign and softmax activation functions.



Scheme for the evaluation of the wave function with RNNs [2].



The 2D RNN structure is designed to prevent a numerical separation of neighboring sites.

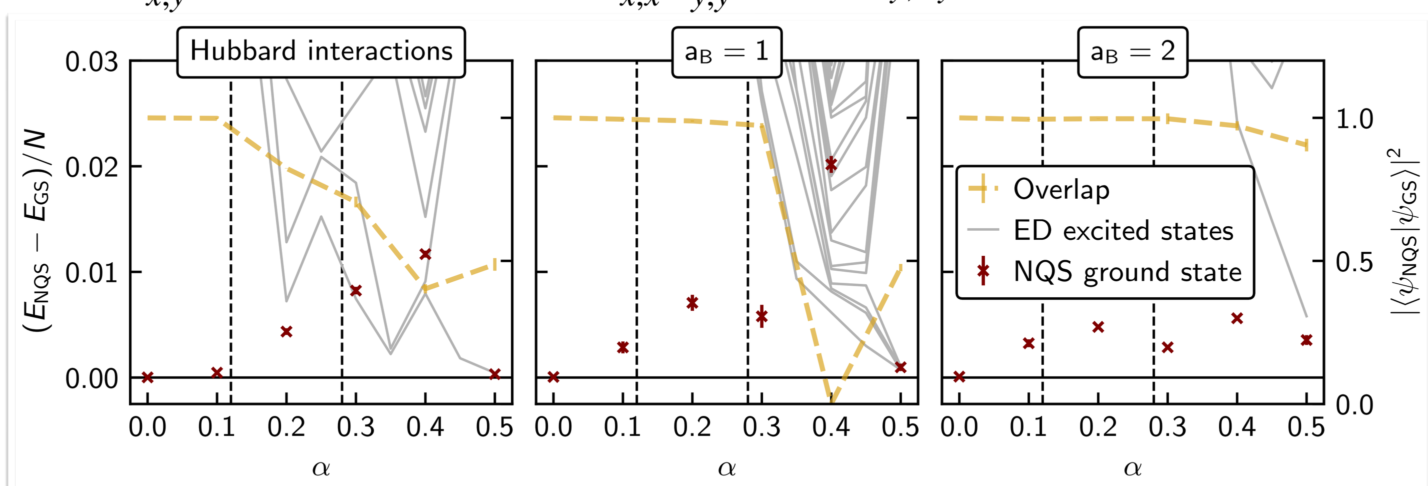


So far, only systems with a naturally restricted local Hilbert space where studied with auto-regressive NQS. To apply this algorithm to bosonic systems, a cut-off has to be introduced.

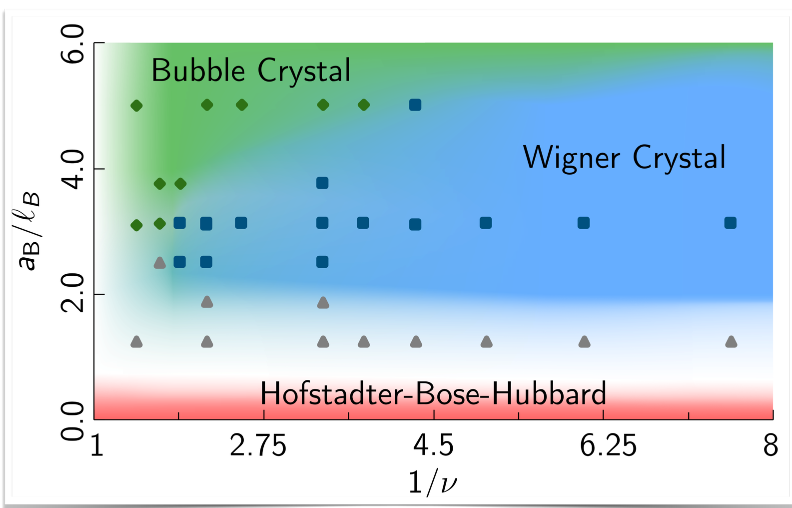
Hofstadter Model with Long-Range Interactions

While most interactions in nature are long-ranged, accurately representing them in numerical simulations can be quite challenging. In this work, we demonstrate that NQS can efficiently model long-range interactions by presenting results for a system of Rydberg-dressed atoms in a magnetic field, which potentially hosts non-Abelian fractional quantum Hall states [5].

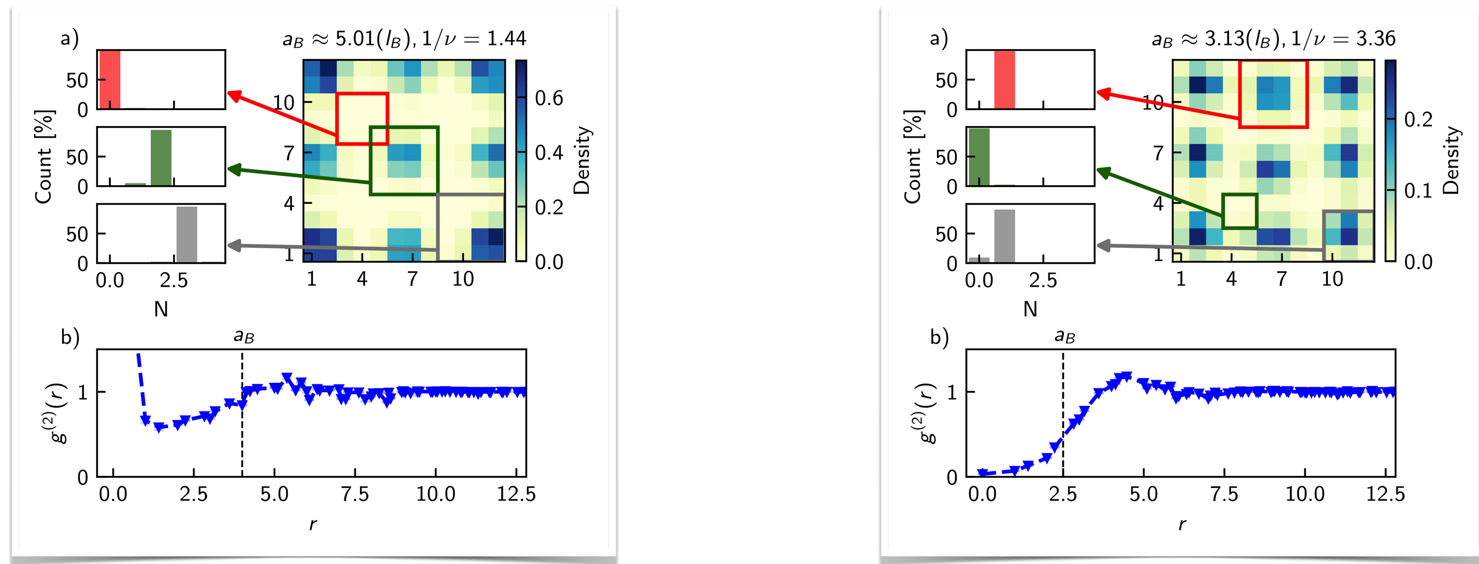
$$\frac{U}{2}\sum_{x,y}\hat{n}_{x,y}(\hat{n}_{x,y}-1) \longrightarrow \frac{U_0}{2}\sum_{x,x'}^{L_x}\sum_{y,y'}^{L_y}\frac{a_B^6}{a_B^6 + r^6_{xy,x'y'}}\hat{n}_{x,y}(\hat{n}_{x',y'} - \delta_{x,x'}\delta_{y,y'}).$$



While the overall energy error for the considered 6×6 system is only marginally affected by the change of interactions, the ground state overlap improved significantly in most cases.



Furthermore, we investigated the phases of a 12×12 lattice system that can be feasible for cold atom experiments, while providing access to novel phases.



Examples for a bubble crystal (left) and a Wigner crystal (right).

References

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