Discovering Local Hidden-Variable Models for Arbitrary Multipartite Entangled States and Arbitrary Measurements

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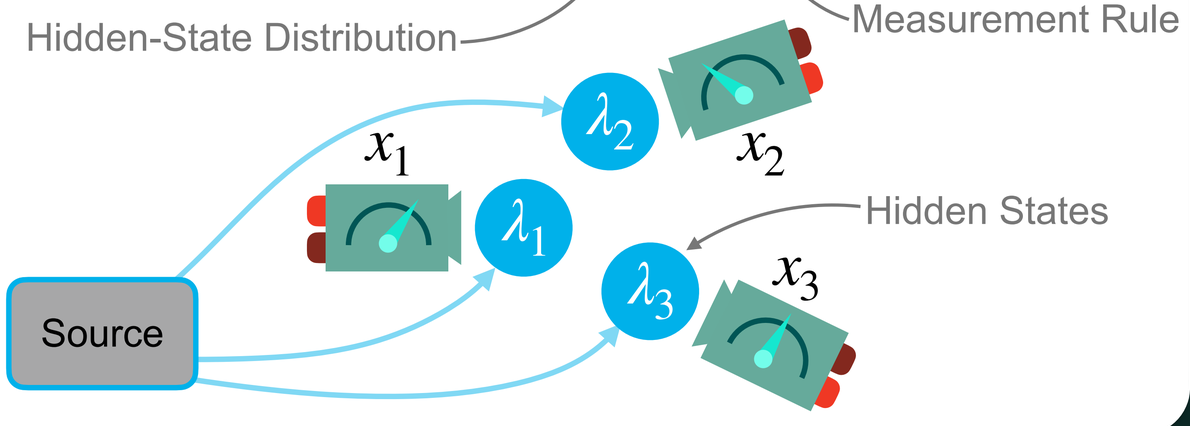
arXiv 2407.04673 GitHub AutoLHVs

Bell Locality [a)]

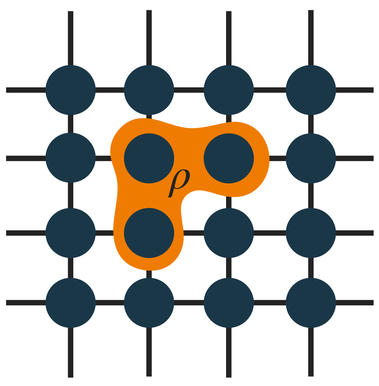
Probability for N parties to measure $\vec{a} = (a_1, \ldots, a_N)$ given their local measurement inputs $\vec{x} = (x_1, \ldots, x_N)$

$$P^{LHV}(\vec{a}\,|\,\vec{x}) = \int_{\Lambda^N} \mathrm{d}\vec{\lambda}\, p(\vec{\lambda}) \prod_{j=1}^{N} q(a_j\,|\,x_j, \lambda_j)$$

Hidden-State Distribution — Measurement Rule



States and Objective



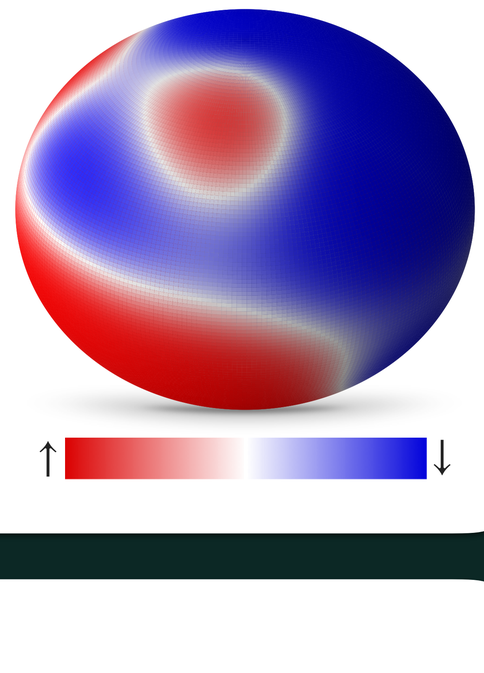
- Optimize LHV models s.t. $P^{LHV} = P^{QM}$
- Minimize loss function $\mathscr{L}(LHV\,||\,QM) \sim$ KL-Divergence
- Noisy entangled states
- Subsystems of many-body states (monogamy of Bell correlations [b)])

Measurement Rule

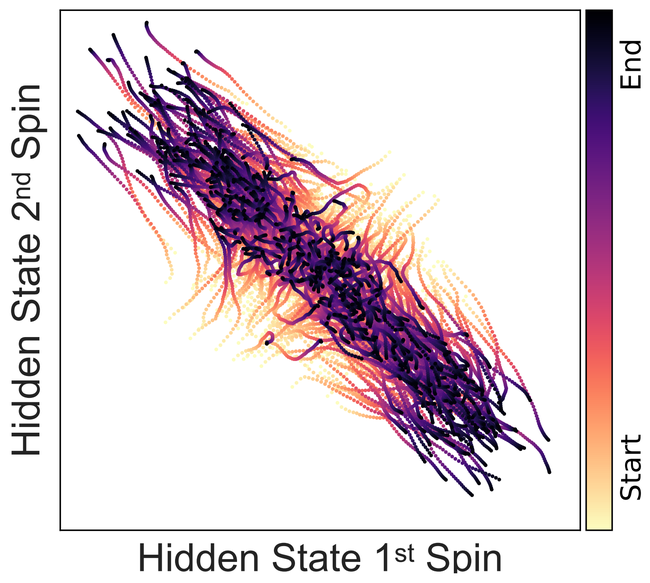
- Spin-1/2 systems
- Probability for "up" along measurement direction $\hat{n} \in S^2$ given a fixed hidden state $\vec{\lambda} \in \mathbb{R}^d$

$$q(\uparrow\,|\,\hat{n}, \vec{\lambda}) = \sigma\left(\vec{S}_{\hat{n}} \cdot \vec{\lambda}\right)$$

- $\vec{S}_{\hat{n}}$: Odd Spherical harmonics
- σ: Sigmoid function



Hidden State Distribution



- Hidden state cloud

$$\left\{\left(\vec{\lambda}_1^k, \ldots, \vec{\lambda}_N^k\right)\right\}_{k=1}^{N_{hidden}}$$

represents the distribution

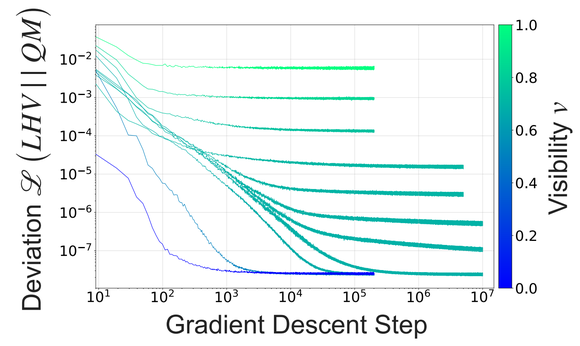
- Optimize via stochastic gradient descent

Werner States [c)]

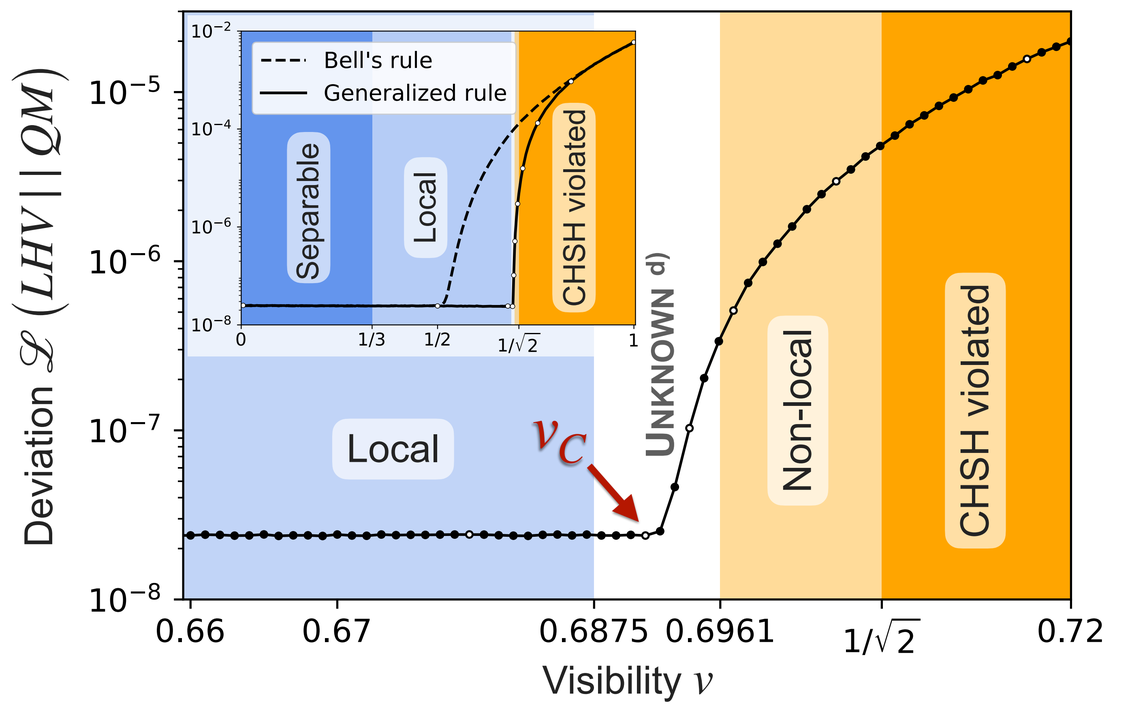
- Tune between Bell singlet and maximally mixed state

$$\rho_v = v\,|\psi\rangle\langle\psi| + (1-v)\frac{\mathbf{1}}{2},$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$



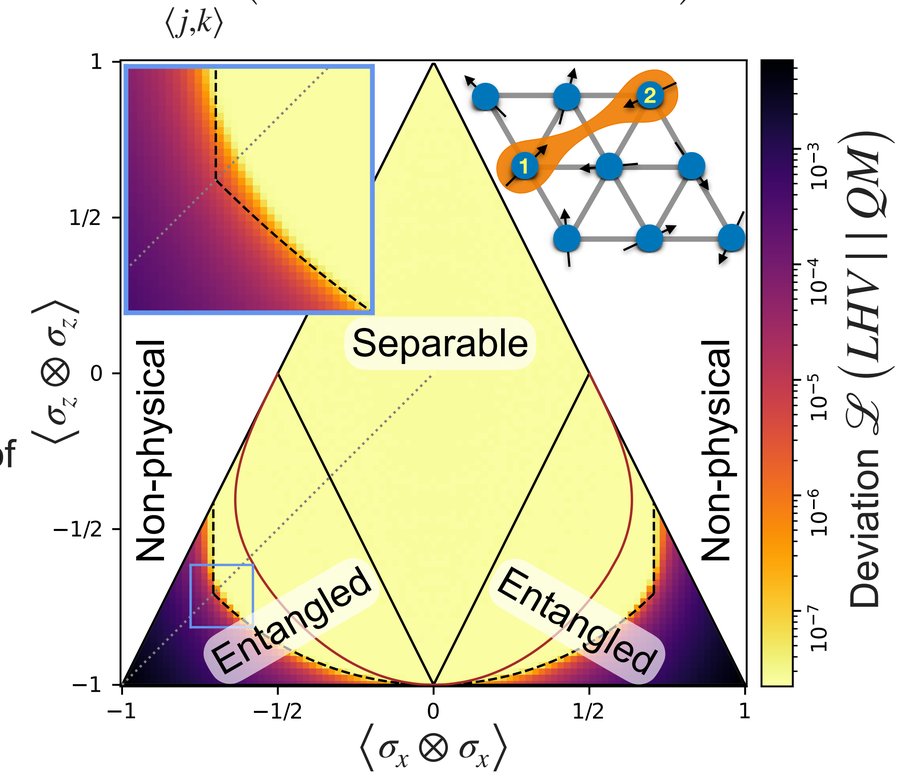
- Local for $v \leq v_C$, entangled for $v > 1/3$

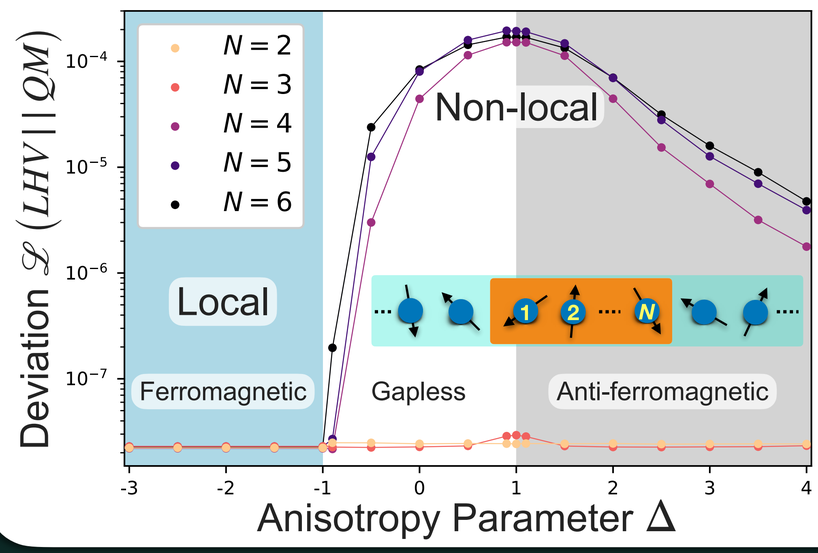


XXZ Models

$$H_{XXZ} = \sum_{\langle j,k\rangle}\left(S_j^x S_k^x + S_j^y S_k^y + \Delta\, S_j^z S_k^z\right)$$

Symmetries restrict two-spin subsystems of GS [e)]





1d chain, N spin nearest neighbour subsystems of GS

References:

[a)] J. S. Bell, Physics Physique Fizika (1964)

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[d)] S. Designolle et al., Physical Review Research (2023)

[e)] F. Verstraete, J. I. Cirac, Physical Review B (2006)