Error-tolerant quantum convolutional neural networks for symmetry-protected topological phases

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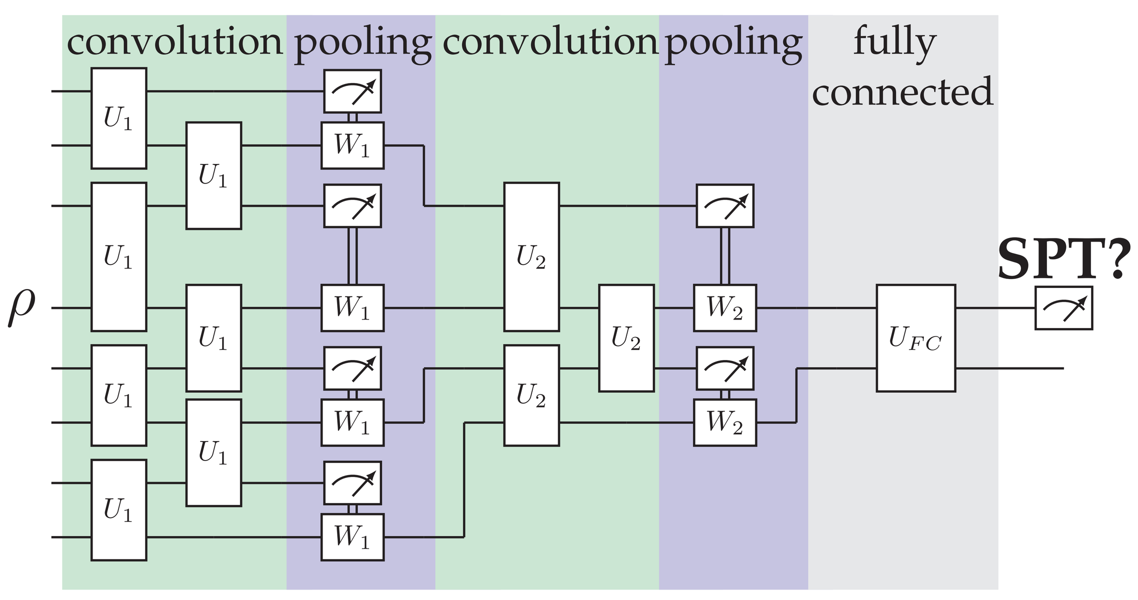
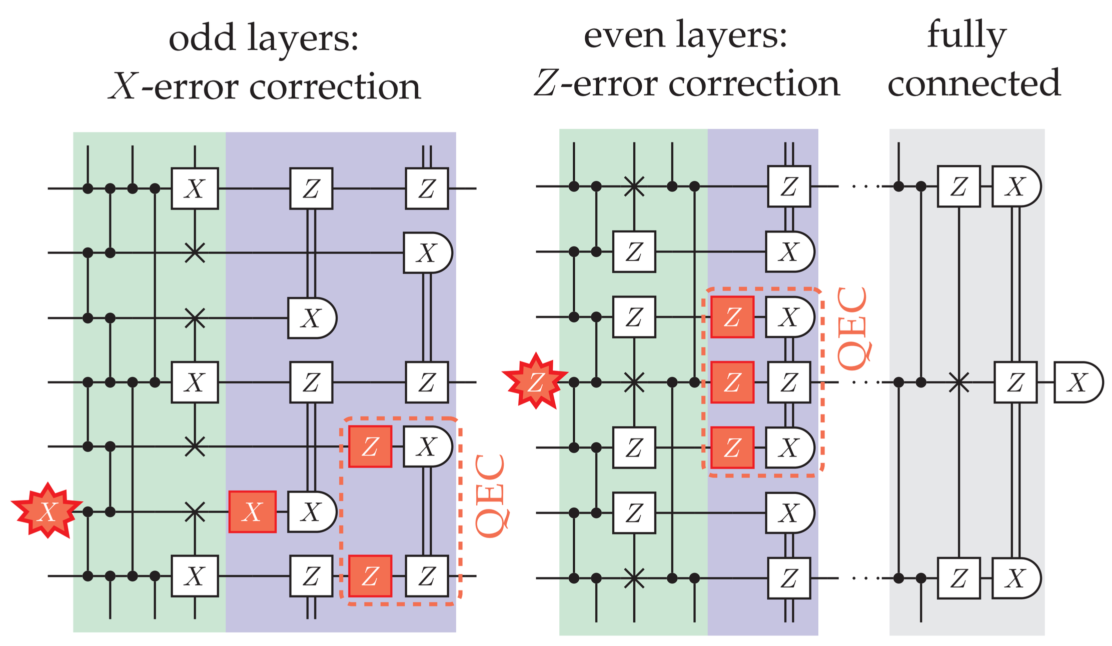
Quantum neural networks based on parametrized quantum circuits, measurements and feedforward can process quantum states produced by quantum computers, to detect non-local quantum correlations with reduced measurement and computational efforts. Here we construct quantum convolutional neural networks (QCNNs) to recognize symmetry-protected topological (SPT) phases in the presence of incoherent errors, simulating the effects of decoherence under NISQ conditions. Using matrix product state simulations, we show that the QCNN output is robust against symmetry-preserving errors, provided that the error channel is invertible, and against symmetry-breaking errors, below a threshold error probability. Even though the error tolerance is limited close to phase boundaries due to a diverging correlation length, the QCNNs can precisely determine critical values of Hamiltonian parameters. To facilitate the implementation of QCNNs, we show how to shorten logarithmic-depth QCNNs to a constant-depth quantum circuit and classical post-processing. The constant-depth circuit reduces sample complexity exponentially with system size in comparison to the direct sampling of the QCNN output using local Pauli measurements.

QCNN



- Motivated by multiscale entanglement renormalization ansatz [1]
- Cluster state $|C^N\rangle$: ground state for N qubits and $h_1 = h_2 = 0$
- Convolutional and pooling (CP) layer: renormalization and error correction $E|C^N\rangle \xrightarrow{\text{CP}} |C^{N/3}\rangle$, for $E = I,\, X_j,\, Z_j$
- Improve the QCNN design of [1] to correct symmetry-breaking Z errors

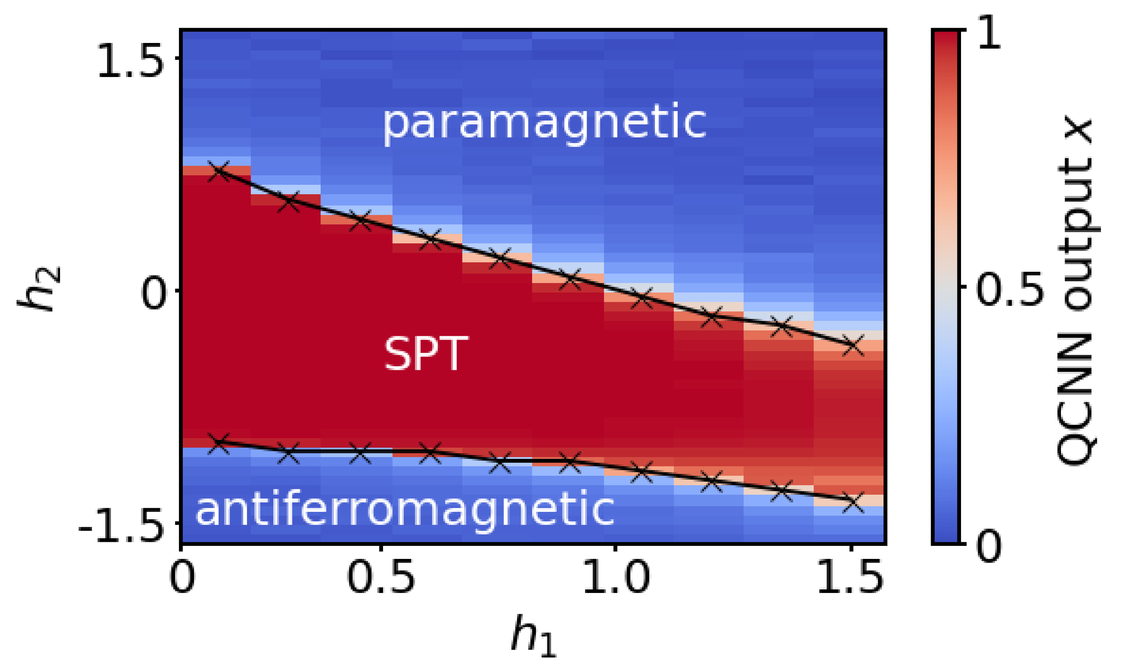


Multiscale SOP

$$\mathcal{S}_{\rm M} = U_{\rm QCNN}^{\dagger} X_i U_{\rm QCNN} = \sum_{jk} \eta^{(1)}_{jk} \mathcal{S}_{jk} + \sum_{jklm} \eta^{(2)}_{jklm} \mathcal{S}_{jk}\mathcal{S}_{lm} + \ldots$$

$\mathcal{O}(16^{3^{d-1}})$ products of SOPs of length $\mathcal{O}(3^d)$ for depth $d = \mathcal{O}(\log_3 N)$

Phase diagram



Depth $d = 4$, $p_X = p_Z = 0.03$

Phase boundary (crosses) by infinite DMRG

Cluster-Ising Hamiltonian

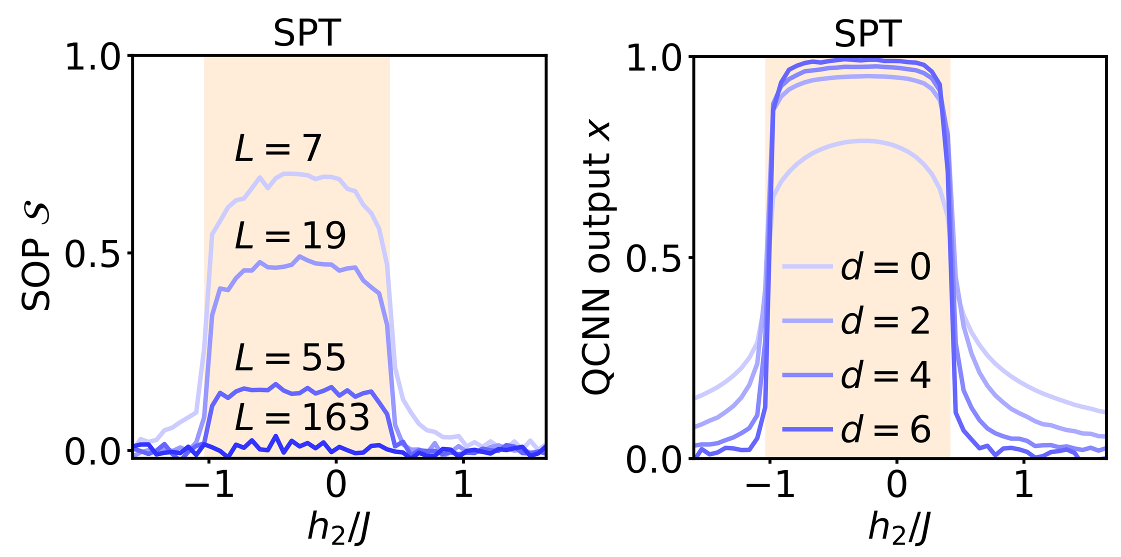
$$H = -\sum_j (J\, C_j + h_1\, X_j + h_2\, X_j X_{j+1})$$

- $C_j = Z_j X_{j+1} Z_{j+2}$, Paulis X_j, Z_j
- $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry-protected topological (SPT) order [2]
- Symmetries $P_{e/o} = \prod_j X_{2j/2j+1}$
- String order parameter (SOP) [3] $S_{jk} = Z_j X_{j+1} X_{j+3} \ldots X_{k-3} X_{k-1} Z_k$

Error tolerance

$$\rho = \mathcal{E}(|GS\rangle\langle GS|) = \sum_{l=0}^{m} K_l |GS\rangle\langle GS| K_l^{\dagger}$$

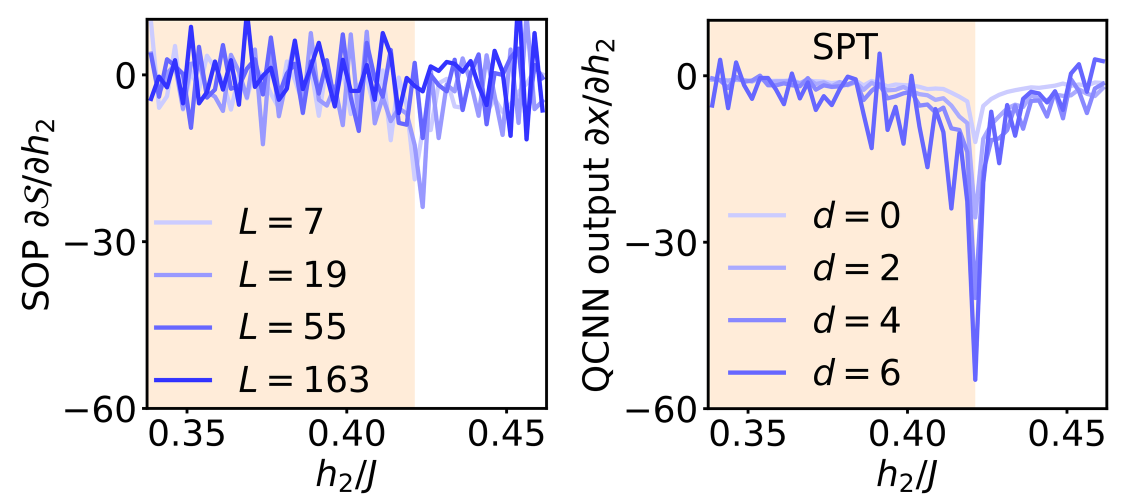
- $K_0 = \sqrt{1-p_E}\mathbb{1}$ and $K_1 = \sqrt{p_E} E$
- NISQ errors $E = X, Z$ with probability p_E due to decoherence
- Exact ground state $|GS\rangle$ (DMRG)



$M_{\rm samples} = 10^4$, $p_X = p_Z = 0.03$, $h_1 = 0.5$

- Symmetry-preserving X errors tolerated provided that $\mathcal{E}$ is invertible
- Symmetry-breaking Z errors tolerated below threshold probability $p_Z = 0.054$
- SOPs vanish for any error probability

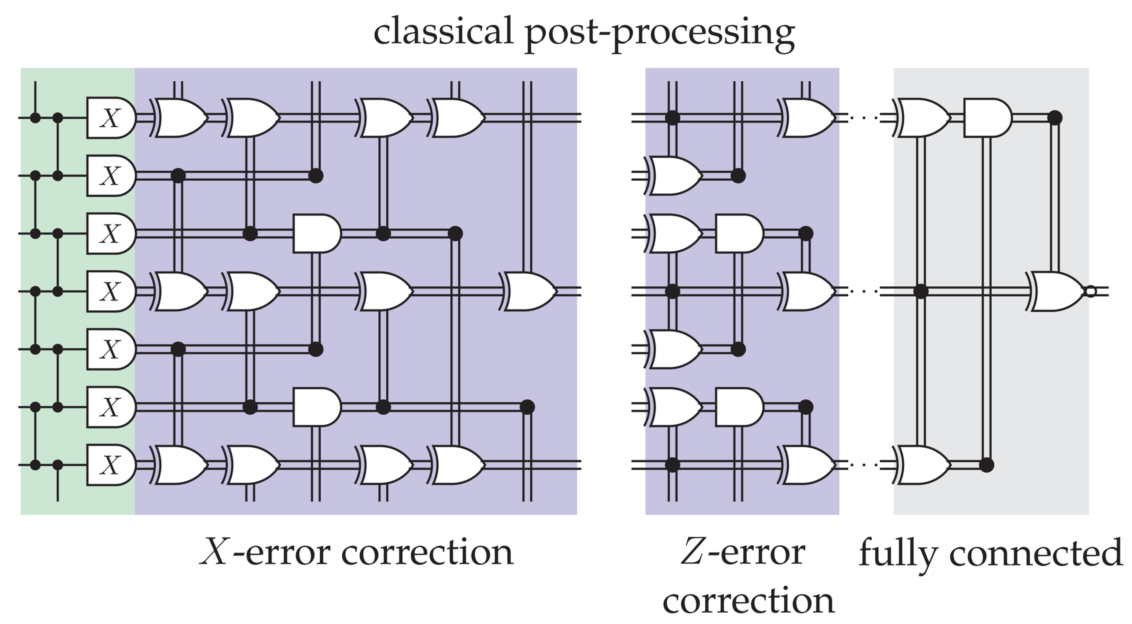
Phase boundary (PB)



$M_{\rm samples} = 10^4$, $p_X = p_Z = 0.03$, $h_1 = 0.5$

- QCNN detects PB as a dip in $\frac{\partial x}{\partial h_2}$

Constant-depth circuit



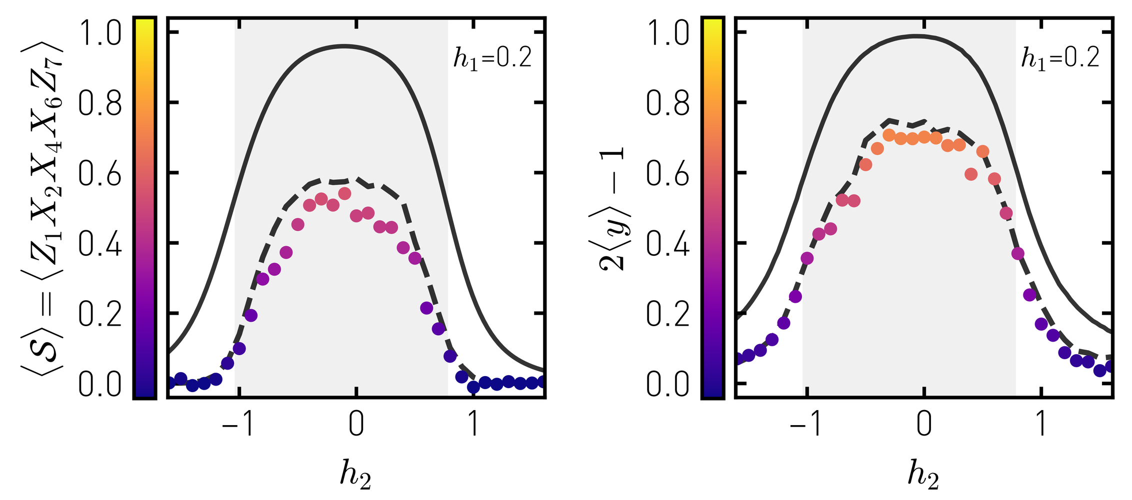
Equivalent QCNN circuit: Two layers of CZ gates and classical post-processing

- Push CZ gates through pooling layers

$$\text{CZ}_{ij}\bar{\text{C}}\bar{\text{C}}\text{NOT}_{kl;j}\text{CZ}_{ij} = \bar{\text{C}}\bar{\text{C}}\text{NOT}_{kl;j}\bar{\text{C}}\bar{\text{C}}\text{Z}_{kl;i}$$

- Exponential reduction of sample complexity due to the remaining CZ gates

Experimental realization [4]



- 7-qubit superconducting processor [4]
- Gates with infidelity $\epsilon = 0.7 - 3.8\%$
- Average infidelity $\tilde{\mathcal{F}} = 0.34$ and $\tilde{\mathcal{F}} = 0.23$ for SOP and QCNN, respectively

Conclusions

- Errors tolerated for $p_Z < 0.054$
- Reduced sample complexity

References

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