Machine Learning Loop Quantum Gravity

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Abstract

In this work, we take the first step of applying deep learning techniques in quantum gravity. We utilise neural network quantum states [1] (NQS) to approach arguably the most difficult and open question in loop quantum gravity (LQG): finding and interpreting solutions to the quantum Hamilton constraints (quantum Einstein equations). We explore 3-dimensional Euclidean gravity [2] in a certain weak coupling limit and rewrite the constraint problem as a ground state search problem by employing the Master constraint program. We consider two regularisations for the constraints and demonstrate that the NQS ansatz can arrive at solutions efficiently and accurately for large Hilbert spaces. We also investigate the similarities and geometric properties of the obtained solutions.

Objectives

- Utilise NetKet [3] for the computational setup
- Employ the NQS ansatz and construct a network architecture robust enough to solve LQG constraints
- Compare the solutions of different regularisations of the quantum Hamilton constraint
- Explore physical properties of the solutions

Physics Results

- NetKet framework robust enough to handle complicated LQG operators
- States near the kernel of both constraints successfully obtained
- The obtained states indicate that the two regularisations describe some common physics
- Investigated properties of geometric operators such as volume

Outlook

- Work in gauge invariant subspaces
- Explore 4d Euclidean gravity in the weak coupling limit
- Characterise the solutions of the constraint
- Release of computational package neuralqx: a package for NQS in LQG built upon NetKet

References

[1] Carleo G. and Troyer M. *Science* 355 602–6, (2017)

[2] Thiemann T. *Class. Quantum Grav.* 15 1249–80, (1998)

[3] Carleo G. et al *SoftwareX* 10 100311, (2019)

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Methodology and Computational Results

1 Hilbert Space

The kinematical Hilbert space is defined on a fixed graph γ. It has a basis labeled by charge vectors $\vec{m}_e \in \mathbb{Z}^3$ defined on the edges of γ. Every charge $m_e^i \in \vec{m}_e$ is allowed to take values in the range $[-m_{max}, \cdots, m_{max}]$.

The vertices constitute "atoms" of space and the edges represent the gluing of those atoms (akin to bonds). Charge labels are quantum numbers of geometry (e.g. area, volume, ...). Vertices carry volume indicated by the shaded plaquettes in the figure. The gluing of such plaquettes is understood as area.

Spin-network functions (SNFs) $\Psi \in \mathcal{H}$ on γ can be understood as a states of space molecule, where the Hilbert space dimensions grows as $\dim \mathcal{H} = (2m_{max}+1)^{3|E(\gamma)|}$.

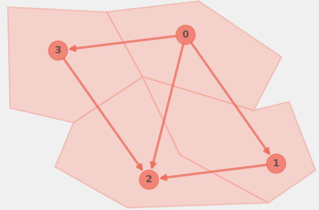


Figure 1: The fixed graph γ used in this work composed of 5 edges and 4 vertices.

m_{max}	$\dim \mathcal{H}$	mem. $\hat{C}$
1	$14\text{x}10^{6}$	1647 TB
2	$30\text{x}10^{9}$	$7.45\text{x}10^{9}$ TB
3	$4.7\text{x}10^{12}$	$1.8\text{x}10^{14}$ TB
4	$2.05\text{x}10^{14}$	$3.39\text{x}10^{17}$ TB

2 Network Architecture

The neural network quantum state (NQS) ansatz parametrises the many-body wave function such that (in the case of the restricted Boltzmann machine architecture) the amplitude in a state describing N-particles each with σ degrees of freedom is given by

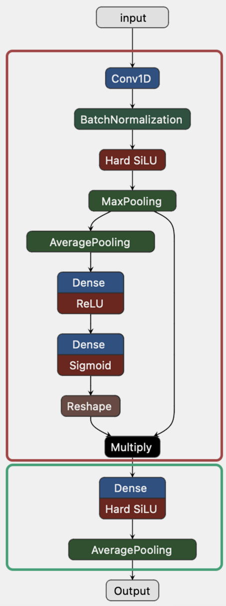
$$\Psi_M(\sigma_1, \cdots, \sigma_N; \{a_i, b_i, W_{ij}\}) = \sum_{\{h_i\}}^{M} e^{\sum_j a_j\sigma_j + \sum_i b_i h_i + \sum_{ij} W_{ij} h_i \sigma_j}$$

where h_i denote the nodes in the hidden layers and $\{a_i, b_i, W_{ij}\}$ are the set of weights. To find a solution for the constraint $\hat{C}$: find all network parameters $\mathrm{Params}(\Psi)$ such that $\min\langle\hat{C}\rangle$ is minimised while $|\mathrm{Params}(\Psi)| < \dim \mathcal{H}$

<u>In this work:</u>

- Architecture employing CNNs
- Attention mechanism and ResNet like skip connections
- Scalable depending on $\dim \mathcal{H}$
- Almost "universal" (different constraints, different graphs)

m_{max}	$\mathrm{Params}(\Psi)$	$\mathrm{Params}(\Psi)/\dim \mathcal{H}$
1	7641	0.053 %
2	6412	$2.101\text{x}10^{-5}$ %
3	6501	$1.369\text{x}10^{-7}$ %
4	23222	$1.127\text{x}10^{-8}$ %



3 Ground State Search

In this weak coupling limit, 3d Euclidean gravity is equivalent to a $U(1)^3$ BF-theory. We quantise it using loop quantum gravity (LQG) methods and investigate two master constraints: $\hat{C} = \hat{F} + \hat{G}$ and $\hat{C}_{TRC} = \hat{H} + \hat{H}^\dagger + \hat{G}$ where

$$\hat{G} = \sum_{v\in V(\gamma)} \left(\sum_{e\in E_{in}(v)} \hat{N}_e - \sum_{e'\in E_{out}(v)} \hat{N}_{e'} \right)^2 , \qquad \hat{F} = \sum_{\alpha\in L(\gamma)} \mathrm{tr}\left[\left(\hat{h}_\alpha - \mathbb{I}\right)\left(\hat{h}_\alpha^\dagger - \mathbb{I}\right) \right]$$

and

$$\hat{H} = \frac{2}{\hbar^2} \sum_{\Delta,\Delta'\in T,v} \epsilon^{ij}\epsilon^{kl} N(v) \mathrm{tr}\left(\hat{h}_{\alpha_{ij}(\Delta')} \hat{h}_{s_k(\Delta)} [\hat{h}^{-1}_{s_k(\Delta)}, \sqrt{\hat{V}_v}] \hat{h}_{s_l(\Delta)} [\hat{h}^{-1}_{s_l(\Delta)}, \sqrt{\hat{V}_v}] \right)$$

is a regulatisation of the quantum Hamilton constraint arising from LQG.

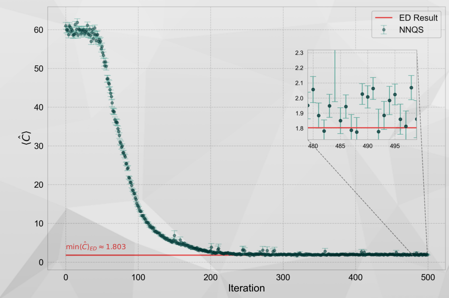


Figure 2: The ground state search simulation for $m_{max} = 2$

Table 1: The results of the ground state search for a solution to the $\hat{C}$ constraint (ED results are estimated).*

m_{max}	$\min\langle\hat{C}\rangle^{(ED)*}$	$\min\langle\hat{C}\rangle^{(NN)}$	Accuracy (%)
1	2.507903	2.998 ± 0.017	80.441
2	1.803495	1.74 ± 0.16	96.286
3	1.168658	1.12 ± 0.11	96.069
4	0.790868	0.84 ± 0.21	93.788

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