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Detection of noise correlations in two ultrastrongly coupled qubits via Machine Learning

Uni ct FISICA E ASTRONOMIA "ETTORE MAJORANA" | LEONARDO

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Abstract: We investigate a machine learning based classification of noise acting on a two qubit system with the aim of detecting noise correlations, and the interplay with Markovianity. We study a ultra strongly coupled two qubit system which can be reduced to an effective three-level system under some particular choice of parameters and approximations. We then control the system using the well known Stimulated Raman Adiabatic Passage (STIRAP) protocol to achieve effective population transfer from ground state to the doubly excited state of the system. We use the efficiency of population transfer under different combinations of external drive amplitudes as inputs to train a feedforward neural network. We show that supervised learning can classify different types of classical noise acting locally on the qubits and affecting the overall system dynamics. In particular as noise models we take into account three non-Markovian (quasi-static correlated, anti-correlated and uncorrelated) and two Markovian noises (correlated and anti-correlated) that are classified with significant accuracy. (Email: shreyasi.mukherjee@dfa.unict.it)

System of interest

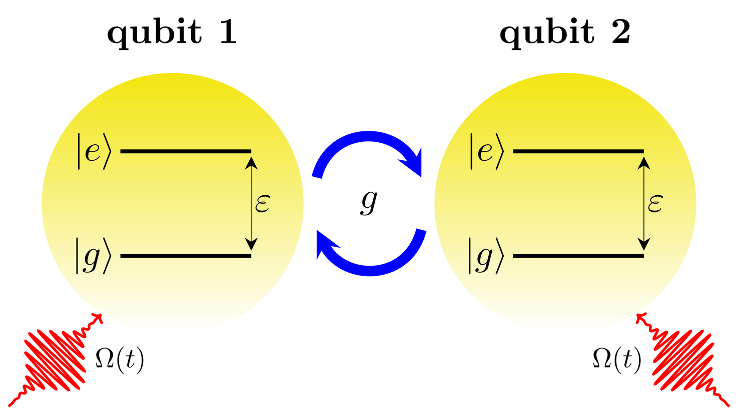
- We consider a system of driven two ultra-strongly coupled qubits subjected to classical noise.
- The total Hamiltonian of the system is: $H = H_0 + H_c + H_{noise}$ where

$$H_0 = -\frac{\varepsilon}{2}(\sigma_1^z \otimes \mathcal{I}_2 + \mathcal{I}_1 \otimes \sigma_2^z) + \frac{g}{2}(\sigma_1^- \sigma_2^+ + \sigma_1^+ \sigma_2^-),$$

$$H_c(t) = \frac{1}{\sqrt{2}}\Omega(t)(\sigma_1^x \otimes \mathcal{I}_2 + \mathcal{I}_1 \otimes \sigma_2^x),$$

$$H_{noise}(t) = -\frac{\tilde{X}_1(t)}{2}\sigma_1^z \otimes \mathcal{I}_2 - \frac{\tilde{X}_2(t)}{2}\mathcal{I}_1 \otimes \sigma_2^z.$$

where $\Omega(t) = \Omega_p(t)\cos(\omega_p t) + \Omega_s(t)\cos(\omega_s t)$



Total Hamiltonian

$$H = \begin{pmatrix} 0 & 0 & \Omega(t) & 0 \\ 0 & 2\varepsilon + \tilde{X}_1(t) + \tilde{X}_2(t) & \Omega(t) & 0 \\ \Omega(t) & \Omega(t) & \varepsilon + \frac{g}{2} & \frac{\tilde{X}_1(t)+\tilde{X}_2(t)}{2} \\ 0 & 0 & \frac{\tilde{X}_1(t)+\tilde{X}_2(t)}{2} & \varepsilon - \frac{g}{2} \end{pmatrix}.$$

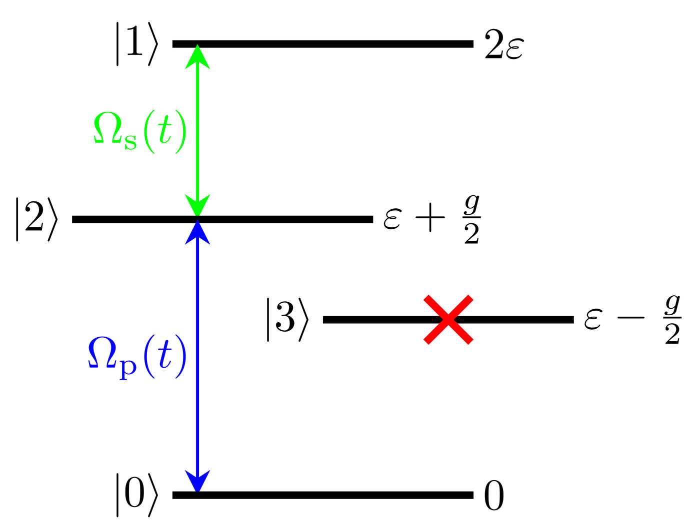
Is population transfer with STIRAP possible in a four-level system?

- In a rotating frame performing RWA and choosing single and two photon resonance condition ($\delta_p = \varepsilon + \frac{g}{2} - \omega_p = 0,\quad \delta_s = \varepsilon - \frac{g}{2} - \omega_s = 0$) respectively and in USC ($g \approx \varepsilon$)

4-level system (without noise)→ 3-level system

$$\tilde{H} = \frac{\Omega_p(t)}{2}|0\rangle\langle 2| + \frac{\Omega_s(t)}{2}|1\rangle\langle 2| + h.c.$$

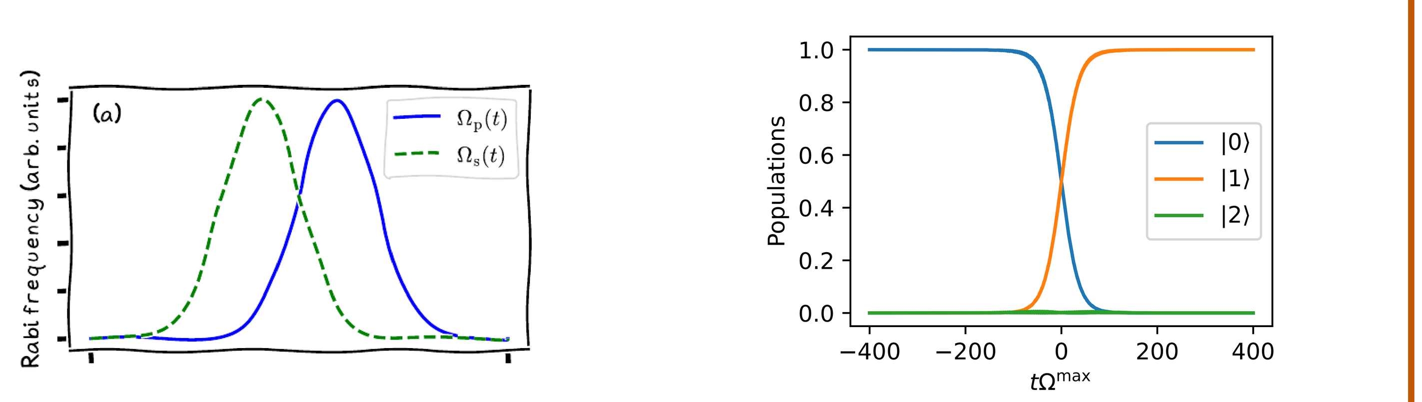
- The noise leads to two-photon detuning, and coupling of level $|3\rangle$. Consequently, the 3-level system picture becomes inaccurate.



Stimulated Ramman Adiabatic Passage (STIRAP)

- adiabatic protocol (efficiency ≈ 1)
- counter-intuitive pulse sequence
- Under two photon deunning condition $\longrightarrow |\phi_D(t)\rangle = \cos\theta(t)|0\rangle - \sin\theta(t)|1\rangle$ (**Dark state**), where $\theta(t) = \tan^{-1}\left(\frac{\Omega_p(t)}{\Omega_s(t)}\right)$.

$$|\psi(t_i)\rangle = |0\rangle \longrightarrow |\psi(t_f)\rangle_{target} = |1\rangle.$$



We point out that our aim is not to achieve efficient population transfer, but to take advantage of STIRAP to get information on the noise that affects the system and deteriorates efficiency.

quasistatic→ noise constant during a single evolution

Noise

Goal is the classification among five kinds of noise with ML

$$H_{noise} = -\frac{\tilde{X}_1(t)}{2}\sigma_1^z \otimes \mathcal{I} - \frac{\tilde{X}_2(t)}{2}\sigma_2^z \otimes \mathcal{I}.$$

- **Non-Markovian quasistatic:**
  1. correlated noise: $x_2(t) = \eta x_1(t)$, with $\eta > 0$,
  2. anti-correlated noise: $x_2(t) = \eta x_1(t)$, with $\eta < 0$,
  3. uncorrelated noise: $x_2(t)$ and $x_1(t)$, are independent from each other.
- **Markovian:**
  - 4a. correlated noise: $x_2(t) = \eta x_1(t)$, with $\eta > 0$,
  - 4b. anti-correlated noise: $x_2(t) = \eta x_1(t)$, with $\eta < 0$.

References:

1. G Falci, M Berritta, A Russo, A D'Arrigo and E Paladino, 2012 Phys. Scr. 2012 014020
2. S Mukherjee, D Penna, F Cirinnà, M Paternostro, E Paladino, G Falci, L Giannelli, arXiv:2405.01987 (2024)

Caculation of efficiency

- **Non-Markovian noise** $\rightarrow i\frac{\partial}{\partial t}|\psi(t)\rangle = H(t)|\psi(t)\rangle$.

single trajectory efficiency $\xi^{(r)} = \xi(\{x_j\}^{(r)}) = |\langle\psi(t_f)|1\rangle|^2$,

$$\text{efficiency } \xi = \int \xi(x_1, x_2)p(x_1, x_2)dx_1 dx_2 \approx \frac{\sum_{r=1}^N \xi(x_1^{(r)}, x_2^{(r)})p(x_1^{(r)}, x_2^{(r)})}{\sum_{r=1}^N p(x_1^{(r)}, x_2^{(r)})}.$$

- **Markovian noise** $\rightarrow \dot{\rho}(t) = -[H_0 + H_c(t), \rho(t)] - \frac{\gamma}{4}(\hat{O}^2\rho(t) + \rho(t)\hat{O}^2 - 2\hat{O}\rho(t)\hat{O})$,

with $\hat{O} = -\tilde{x}_1(t)(\sigma_1^z \otimes \mathcal{I} + \eta\mathcal{I}_1 \otimes \sigma_2^z)$, $\langle\tilde{x}_1(t)\tilde{x}_1(t')\rangle = \gamma\delta(t-t')$ and $\langle\tilde{x}(t)\rangle = 0$.

$$\text{efficiency } \xi = \text{Tr}\{|1\rangle\langle 1|\rho(t_f)\}.$$

Dataset generation

$$\Omega_p(t) = \Omega_p^{(max)} e^{-\left(\frac{t-\tau}{T}\right)^2},$$
$$\Omega_s(t) = \Omega_s^{(max)} e^{-\left(\frac{t+\tau}{T}\right)^2}.$$

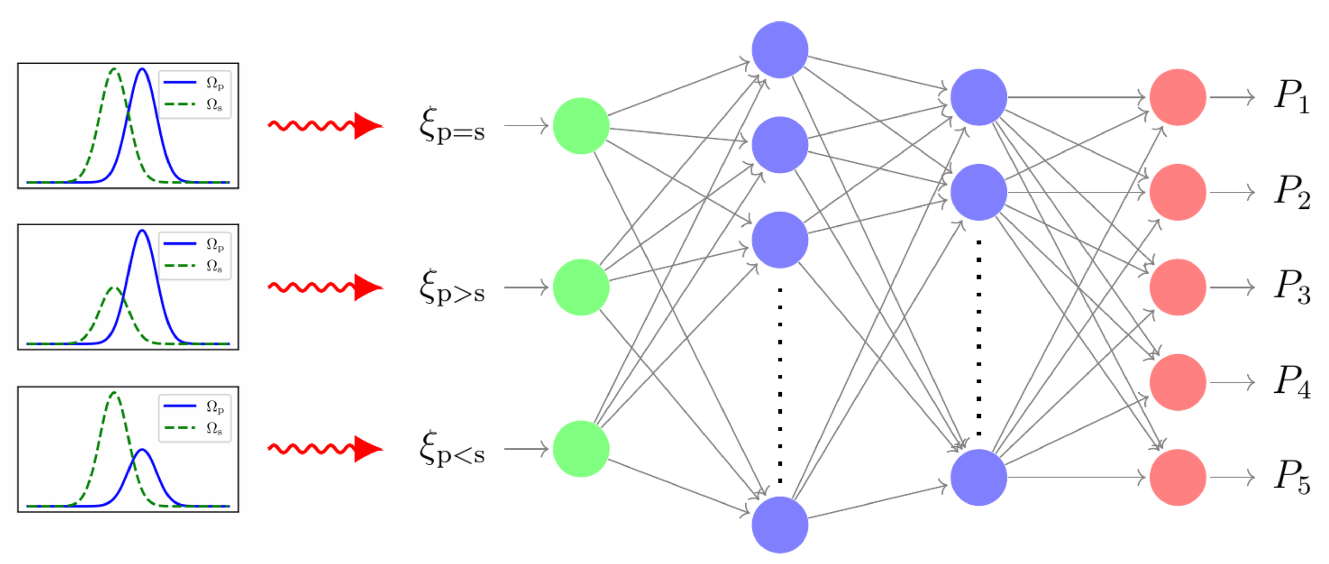
where $\tau = 0.7T$

$\Omega_p^{(max)}, \Omega_s^{(max)} \longrightarrow$ maximum pulse amplitudes.

We assess efficiency under three conditions [1]:

1. $\Omega_p^{(max)} = \Omega_s^{(max)}$ $(0.05\epsilon, 0.05\epsilon)$,
2. $\Omega_p^{(max)} > \Omega_s^{(max)}$ $(\sqrt{0.004}\epsilon, \sqrt{0.001}\epsilon)$,
3. $\Omega_p^{(max)} < \Omega_s^{(max)}$ $(\sqrt{0.001}\epsilon, \sqrt{0.004}\epsilon)$,

Input to our model!



Methodology

1. randomly choose η(correlated) / σ (uncorrelated) / γ(Markovian)
2. calculate efficiency under the three above conditions
3. repeat

- **Non-Markovian quasistatic:**
  1. correlated noise: 500 samples with $\eta \in [0.1, 5]$,
  2. anti-correlated noise: 500 samples with $\eta \in [-5, -0.1]$,
  3. uncorrelated noise: 500 samples with $\sigma_1, \sigma_2 \in [0.002, 0.05]\epsilon$.
- **Markovian:**
  - 4a. correlated noise: 500 samples with $\eta \in [0.1, 5]$, $\gamma \in [0.0001, 0.001]\epsilon^{-1}$,
  - 4b. anti-correlated noise: 500 samples with $\eta \in [-5, -0.1]$, $\gamma \in [0.0001, 0.001]\epsilon^{-1}$.

Results

- **two-qubits (4 -level system)**

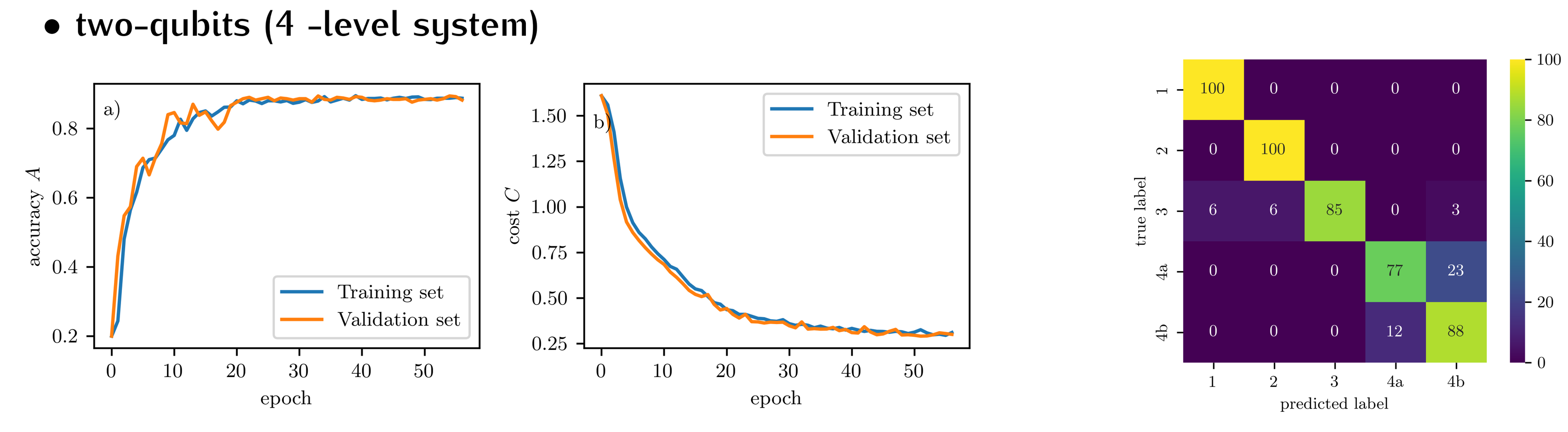


Figure 1: (a) Accuracy and (b) value of the cost function, for the validation (orange) and training (blue) sets versus the number of epochs of training. The accuracy on the test set is $A \approx 0.9$.

- **3 -level system** [2]

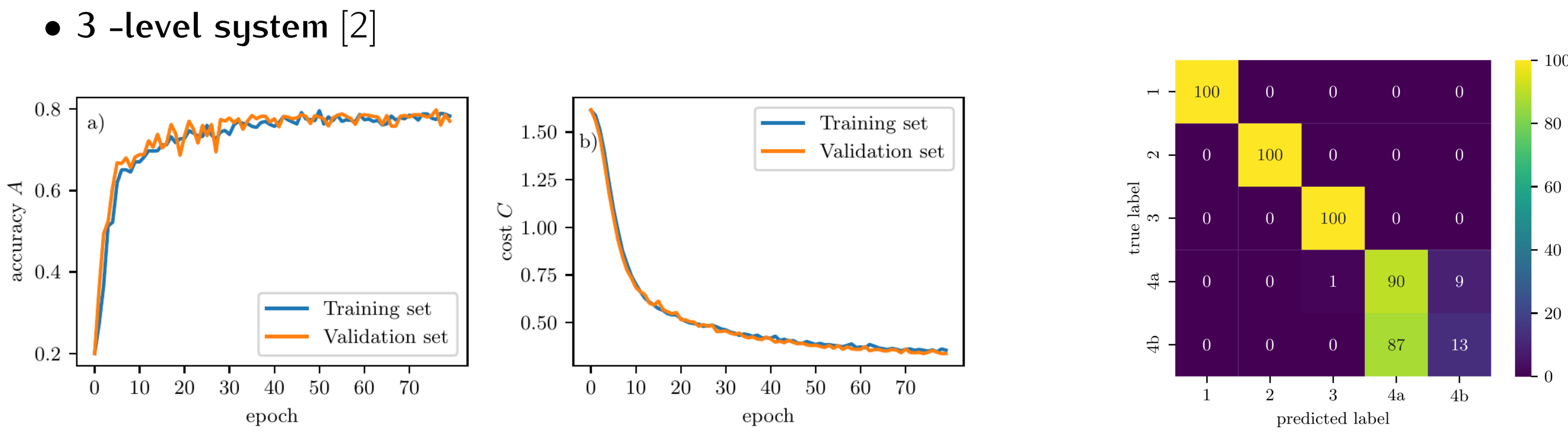


Figure 2: (a) Accuracy and (b) value of the cost function, for the validation (orange) and training (blue) sets versus the number of epochs of training. The accuracy on the test set is $A \approx 0.81$.

- with 4-level system it is also possible to discriminate Markovian noise classes.