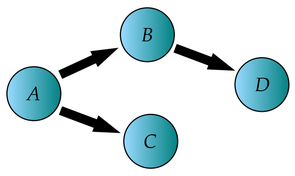
Algorithms for Quantum Causal Discovery

Christina Giarmatzi[1], Jasleen Kaur[1,2], Abhinash Kumar Roy[1,2], Fabio Costa[3] and Alexei Gilchrist[1,2]
[1]Department of Physical and Mathematical Sciences, Macquarie University, Sydney NSW, Australia
[2]Centre for Engineered Quantum Systems, Macquarie University, Sydney NSW, Australia
[3]Nordita, Stockholm University and KTH Royal Institute of Technology, Stockholm, 106 91, Sweden

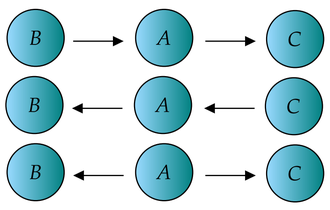
Introduction



What affects what and how?

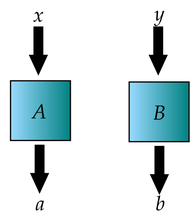
Aim: 1. To find causal influences among the labs.
2. To find mechanism behind influence.

Correlation does not imply causation!



All these causal mechanisms are equivalent on the level of correlations, i.e., $p(a,b,c)$ is the same.

Signalling



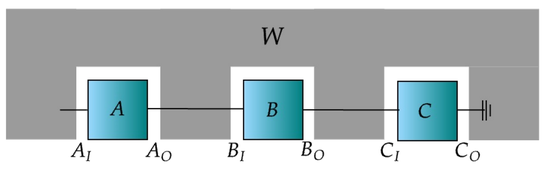
x, y – *settings of labs*
a, b – *outcomes of labs*

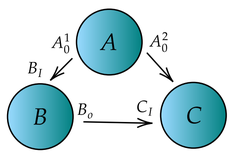
$p(a|x,y) \stackrel{?}{=} p(a|x)$

If yes, $\Rightarrow B \not\prec A$.

Process matrix framework

W represents a process connecting the parties and can be reconstructed using tomography.

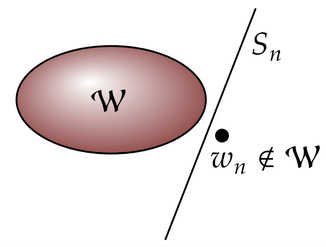




A DAG representing a quantum causal model.
Two channels connecting the parties: $T^{A_O^1 B_I}$ and $T^{A_O^2 B_O C_I}$

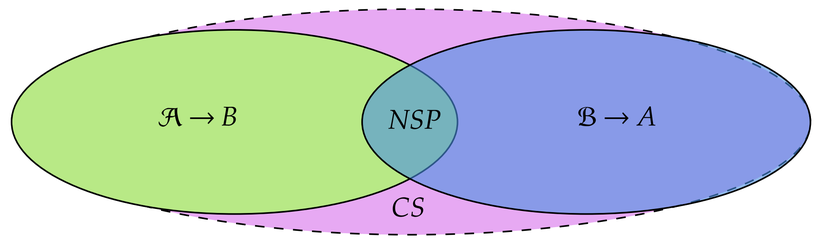
Causal order witness

- A Hermitian operator S is a witness of set $\mathcal{W}$ if $Tr[SW] \geq 0 \; \forall \; W \in \mathcal{W}$
- Hyperplane separation theorem:
  For $W_n \notin \mathcal{W} \; \exists \; S_n$ such that $Tr[S_n W_n] < 0$



Algorithm:

1. Consider a causally ordered set $\mathcal{W}$.
2. Guess a process W_g.
3. Search over the set of witnesses to obtain a witness S_g such that $W_g \notin \mathcal{W}$ using SDP.
4. Measure the witness experimentally.



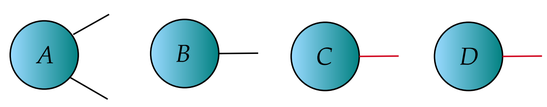
Output: Negative overlap $\Rightarrow W_{exp}$ is not compatible with set $\mathcal{W}$.

Informationally complete regime

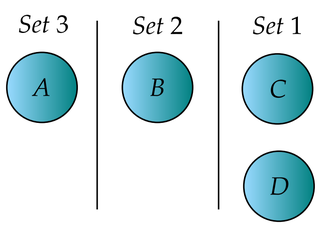
Input: 1. Number of parties.
2. Dimension of each input and output systems.
3. Process matrix.

Algorithm:

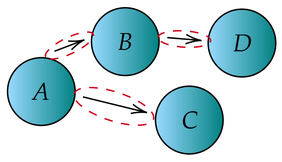
1. Identify open ends. $W = Tr_X[W] \otimes \mathbb{I}^X / d_X$ where $X = C_O, D_O$



2. Identify causal sets.



3. Identify causal arrows.



$\mathrm{Tr}_I W = \mathrm{Tr}_O(\mathrm{Tr}_I W) \otimes \frac{\mathbb{I}^O}{d_O}$
where, $I = D_I, B_I, C_I$ and $O = B_O, A_{O_1}, A_{O_2}$ respectively.

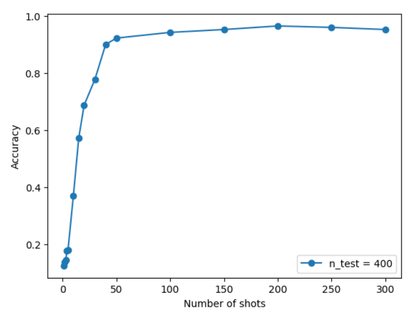
4. Test for Markovianity: $W \stackrel{?}{=} W^{A_I} \otimes W^{A_{O_1} B_I} \otimes W^{A_{O_2} C_I} \otimes W^{B_O D_I} \otimes \mathbb{I}^{C_O D_O}$

Output: $W = \rho^{A_I} \otimes T^{A_{O_1} B_I} \otimes T^{A_{O_2} C_I} \otimes T^{B_O D_I} \otimes \mathbb{I}^{C_O D_O}$

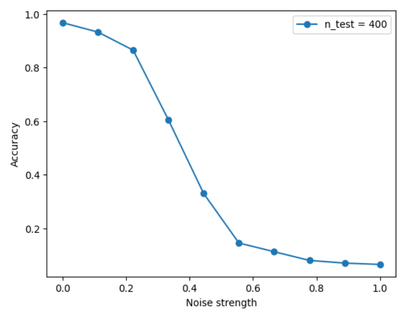
Causal order using AI

- **Problem**: Multiclass classification: thirteen possible labels (different signalling in three party processes)
- **Model description**: Artificial Neural Network
- **Features**: Counts corresponding to informationally incomplete set of operations; model with 98% accuracy with 200 shots per instrument.

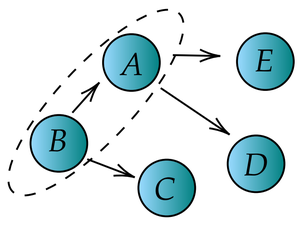
Model accuracy with multinomial noise



Model accuracy with Gaussian noise



Reduced process approach



Given a large process matrix W^{ABCDE}, the causal discovery algorithm as well as SDP becomes inefficient. To find signalling in such cases, find reduced processes with two party at a time as $W^{AB} = Tr_{CDE} W^{ABCDE}$ and use any of the above methods.

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