# Unitary compilation using the Quantum Wasserstein distance


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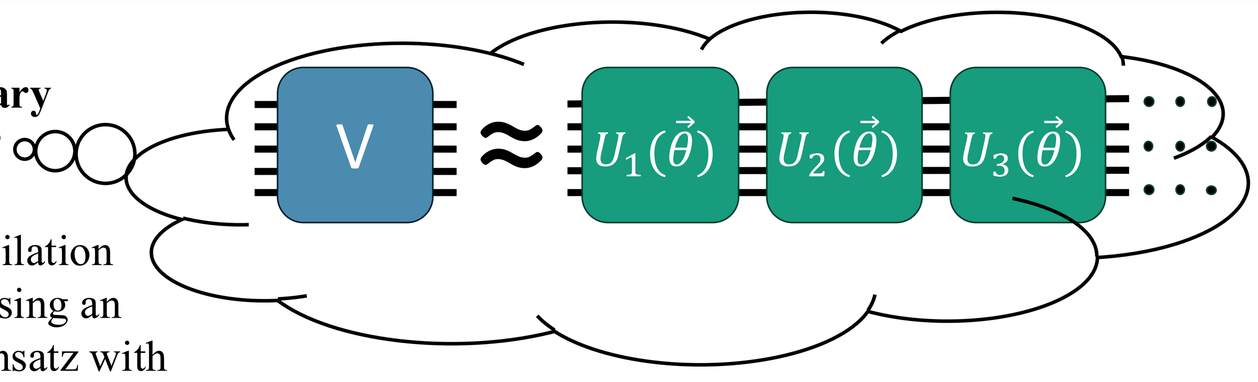
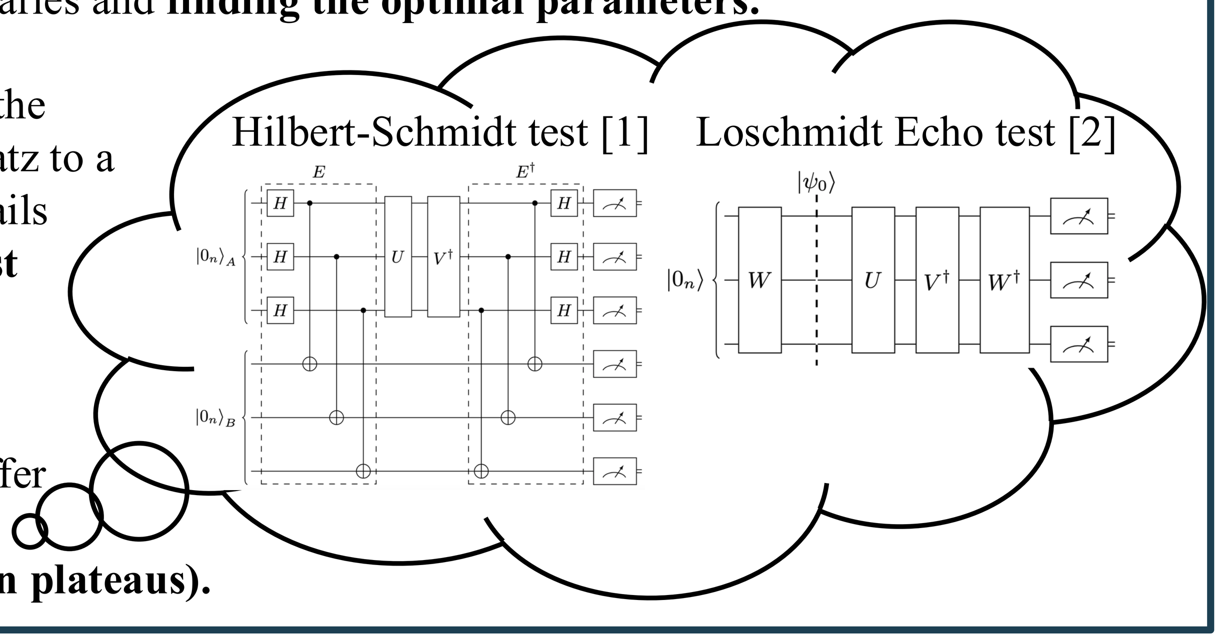
Our Research

## Unitary compilation overview

- What is **Unitary compilation?**




- Unitary compilation requires choosing an appropriate ansatz with parametrized unitaries and **finding the optimal parameters.**
- The closeness of the parametrized ansatz to a target unitary entails **minimizing a cost function**.




- Commonly used cost functions suffer from **vanishing gradients (barren plateaus).**

## Proposed Method

- De Palma *et al.* [3] introduced the **Wasserstein distance of order 1** for quantum states (quantum $W_1$ distance).
- Kiani *et al.* [4] used this distance in a **quantum Generative adversarial net** to learn a quantum state, demonstrating no barren plateaus effects.
- The following inequality holds between the **ideal quantum Wasserstein distance and the average fidelity**

$$C_{QW}(U,V) = \int_\psi d\psi\; W_1^2(U|\psi\rangle, V|\psi\rangle) \geq 1 - \bar{F}(U,V)$$

- Empirically, **we restrict the number of states and the number of Pauli observables** and re-define the cost function.

$$\tilde{C}_{QW}^{(k)}(U,V,\mathscr{A}) = \frac{1}{|\mathscr{A}|}\sum_{\psi\in\mathscr{A}} \left(W_1^{(k)}(U|\psi\rangle, V|\psi\rangle)\right)^2$$

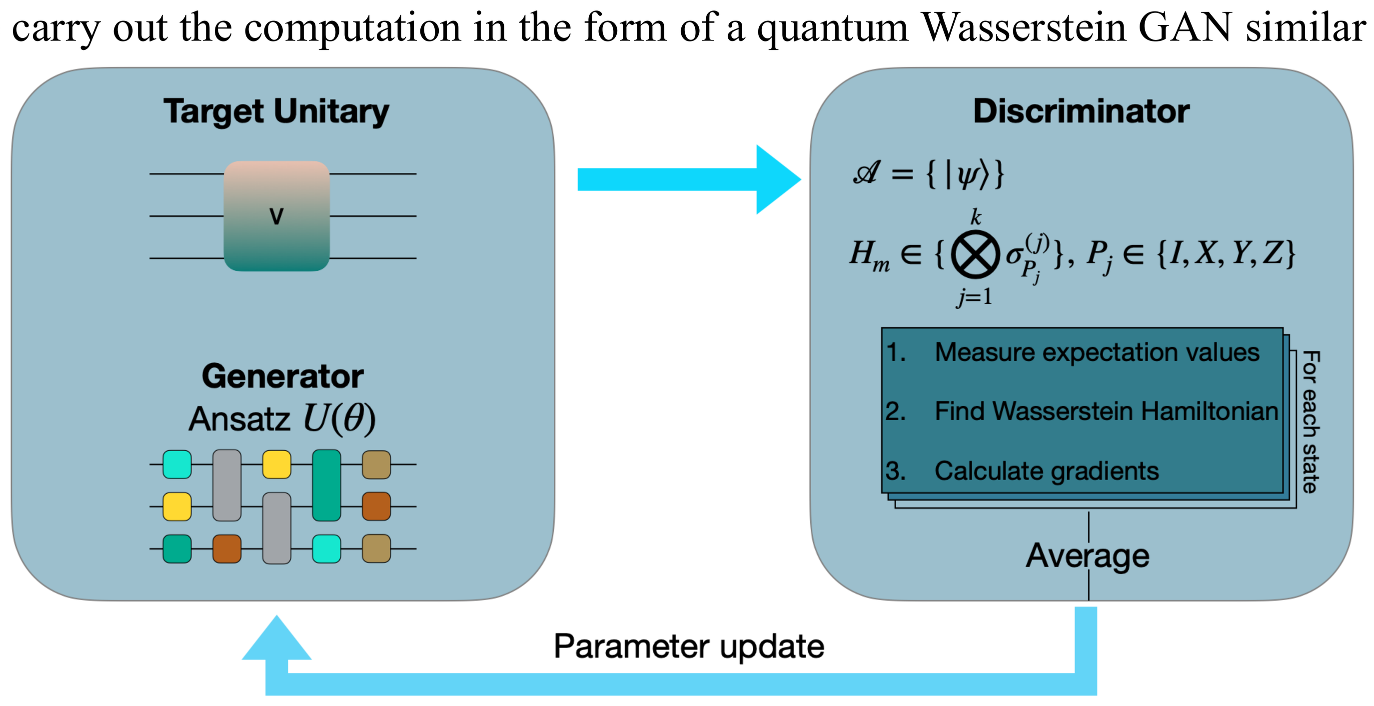
- We calculate the **expectation values of the state from the generator and the target** and construct the Wasserstein Hamiltonian.

$$W_1^{(k)} = \max(\mathrm{Tr}[H(\rho-\sigma)] : ||H||_L < 1)$$

- The **Hamiltonian is a linear combination of $k$-Pauli strings** with $k < n$.

$$H = \sum_m w_m H_m,\quad H_m = \bigotimes_{j=1}^{k} \sigma_{P_j}^{(j)},\quad P_j \in \{I,X,Y,Z\}$$

- We carry out the computation in the form of a quantum Wasserstein GAN similar to [4]



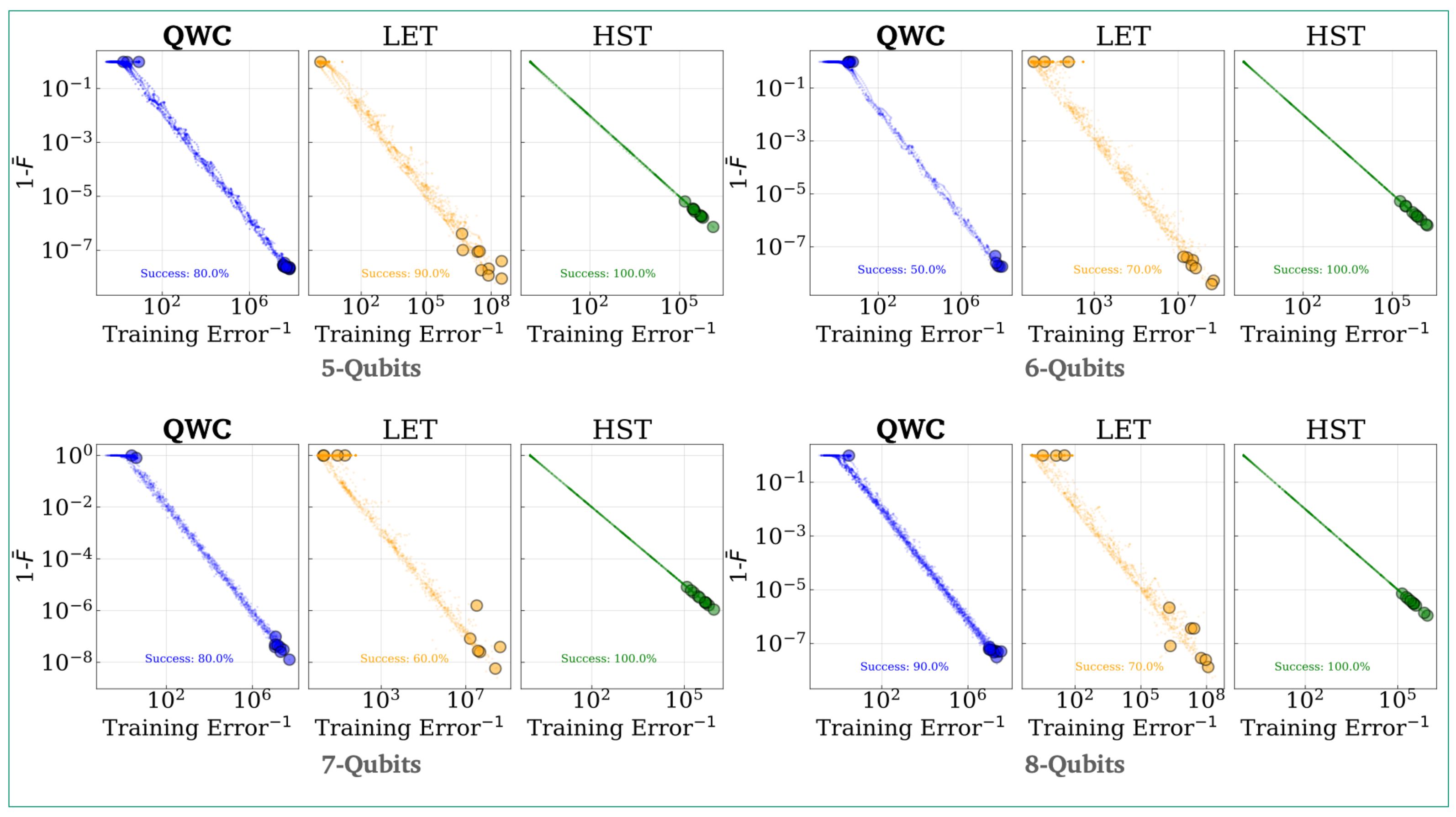

## Problem

**IS THERE A COST FUNCTION FOR VQC THAT IS NOT AFFECTED BY BARREN PLATEAUS?**

## Results & Discussions

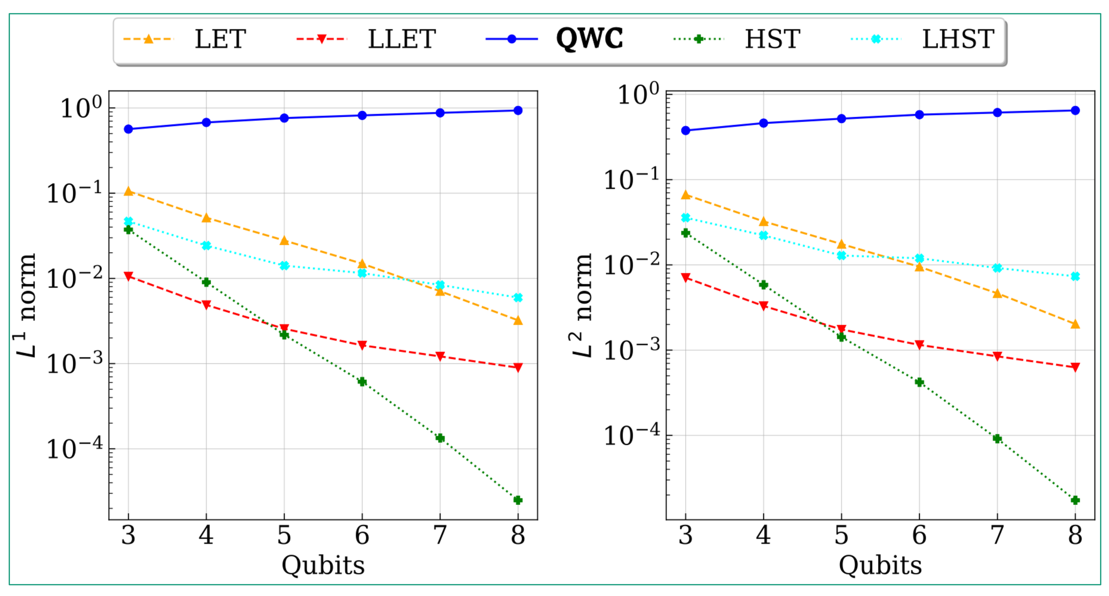
### Infidelity vs. Inverse cost function

- The results show that reducing the cost function guarantees reducing the average infidelity between the states.
- We use a fixed number of states ($|\mathscr{A}| = 8$) in all the experiments.




### One-step gradients for different qubits

- Using the quantum $W_1$ distance mitigates the presence of barren plateaus.
- LET and HST are affected drastically as we scale to higher qubits.




## Outlook

- More efficient methods for Wasserstein distance estimation.
- The scaling of measurement observables needs to be reduced for larger qubit counts.
- Noise resilience of the defined cost function.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