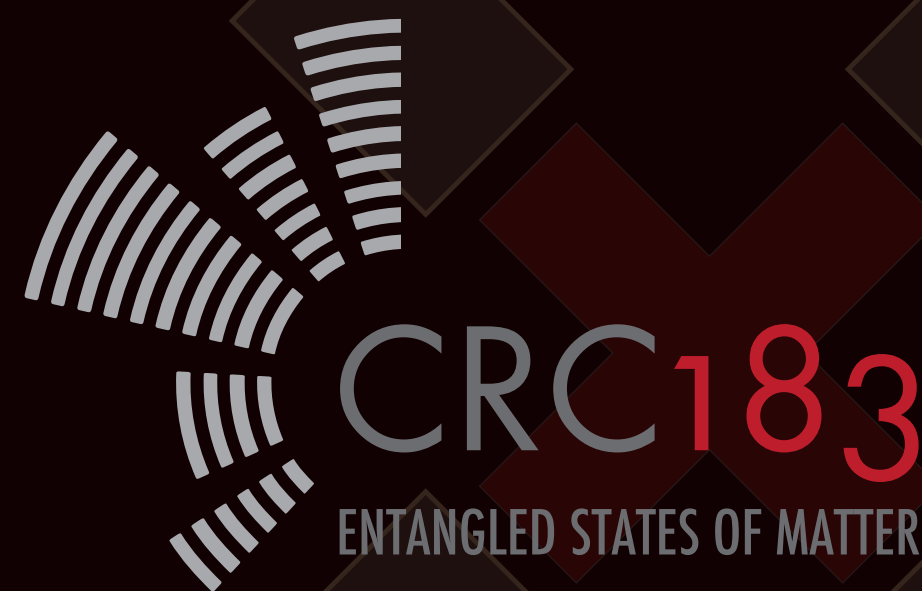


Decoding quantum many-body teleportation

robust teleportation | weak measurements | wave function deformations



Simon Trebst
University of Cologne



2nd Workshop on Machine Learning for Quantum Technology

Max Planck Institute for the Science of Light, Erlangen, November 2024

quantum measurements

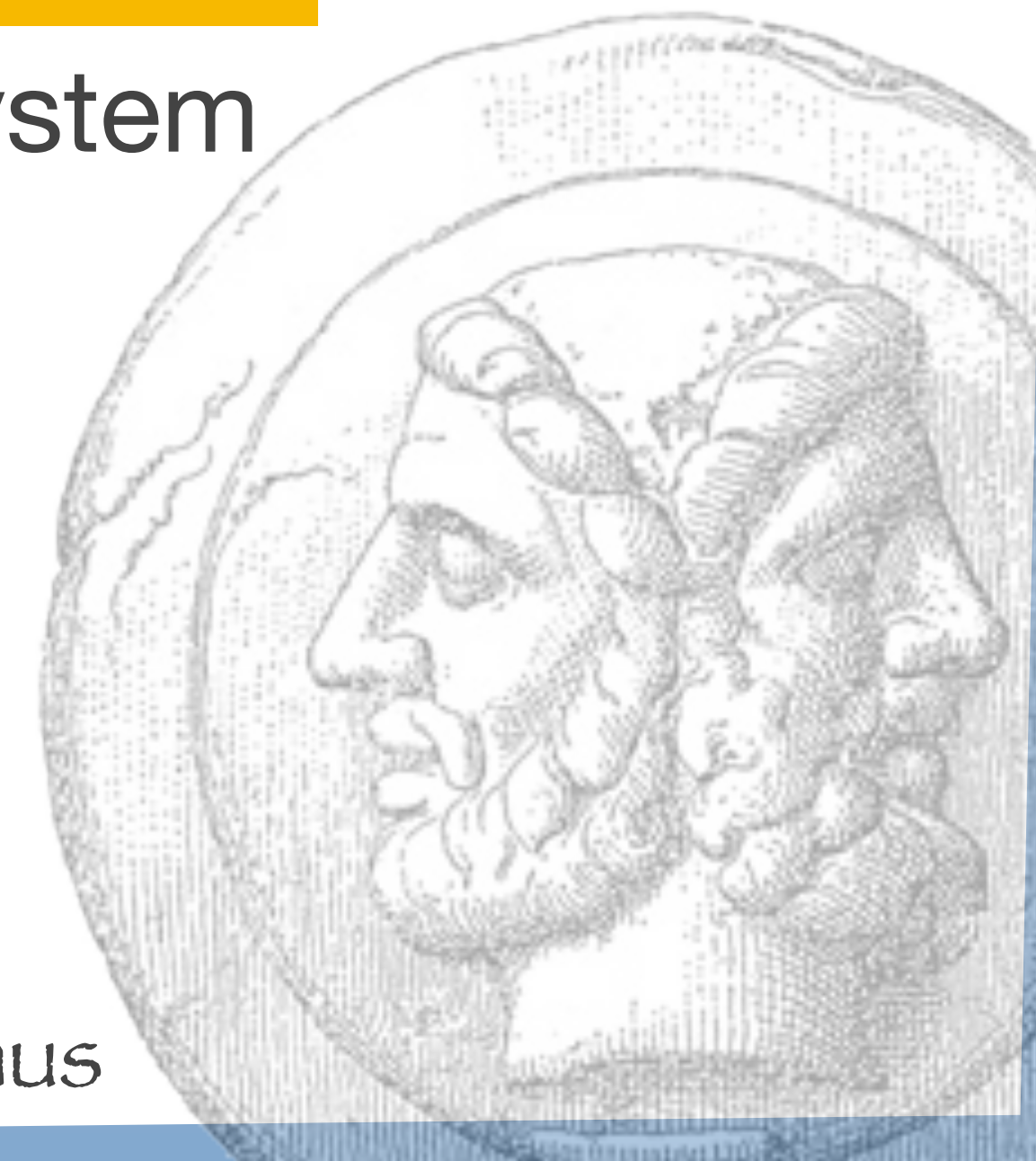


Quantum measurements can

- **extract information** from a system
- **shape entanglement** of a quantum system

“About your cat, Mr. Schrödinger — I have good news and bad news.”

double-faced Janus

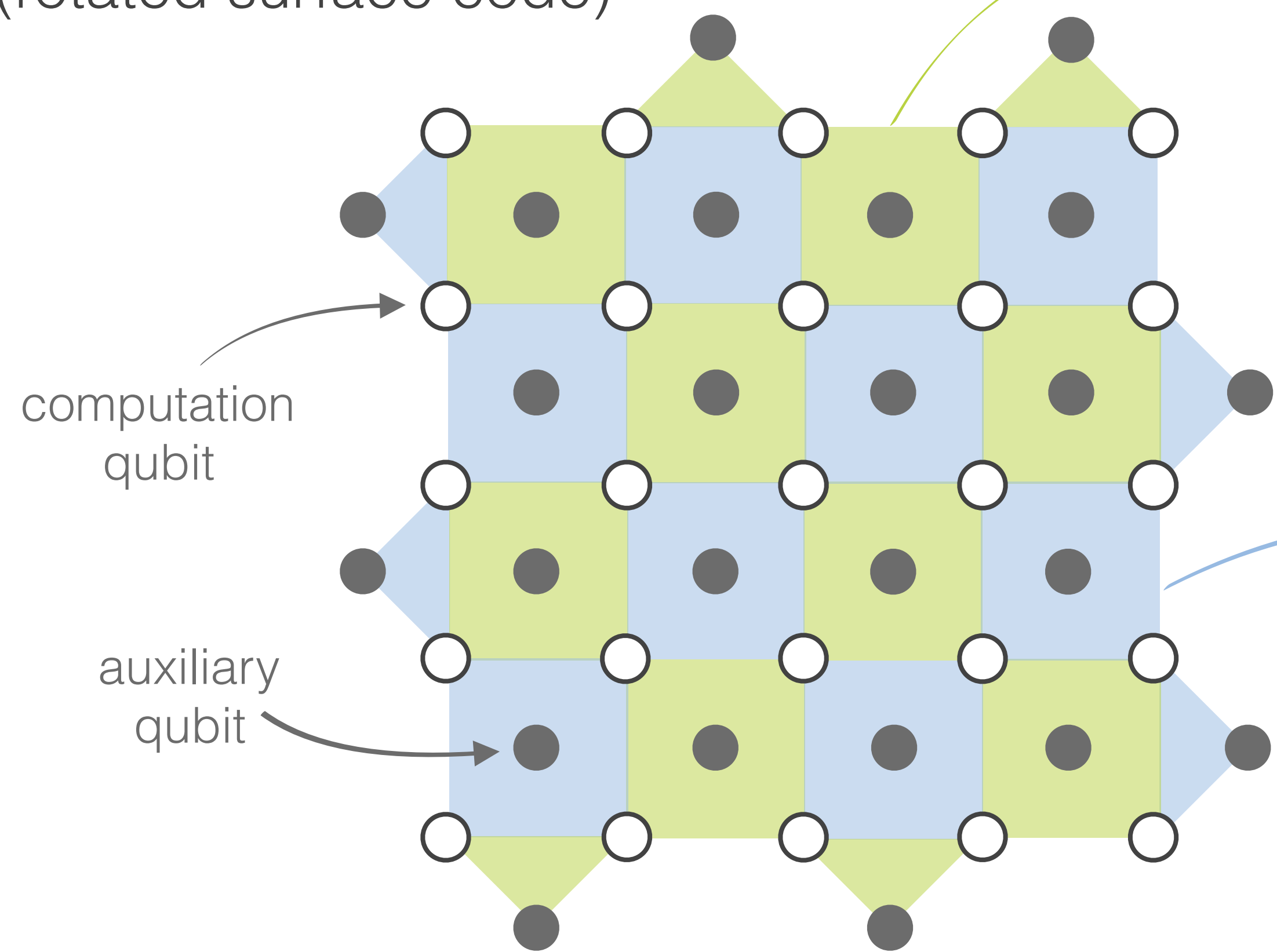


paradigmatic example: **surface code**



Kitaev (1997)

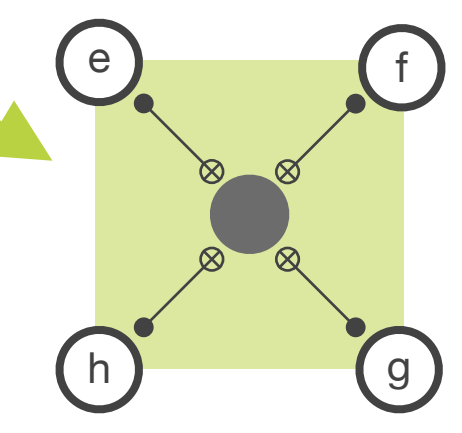
open boundaries
(rotated surface code)



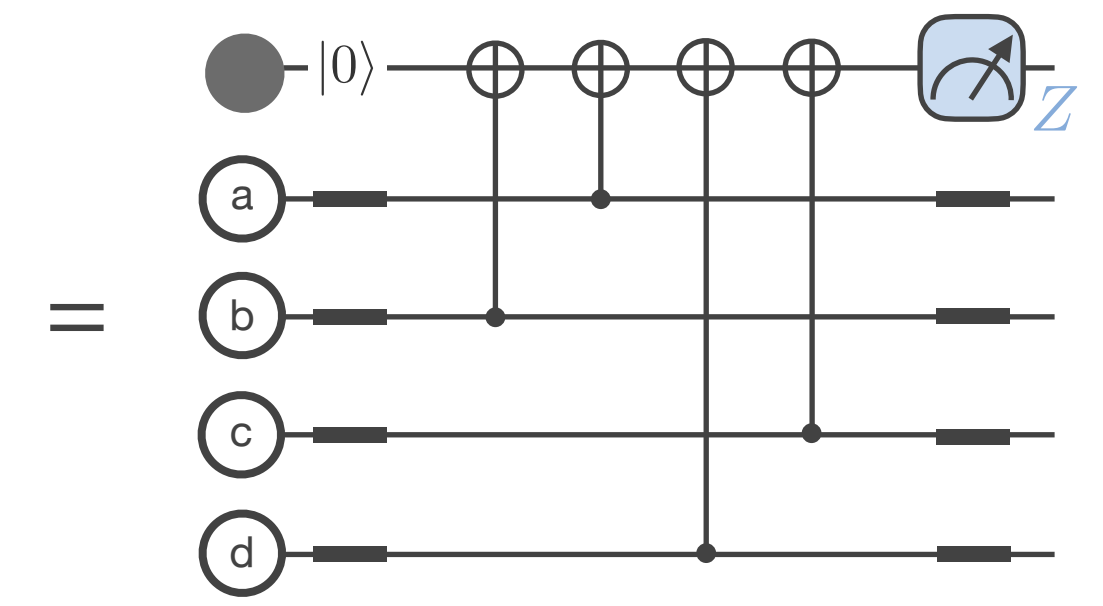
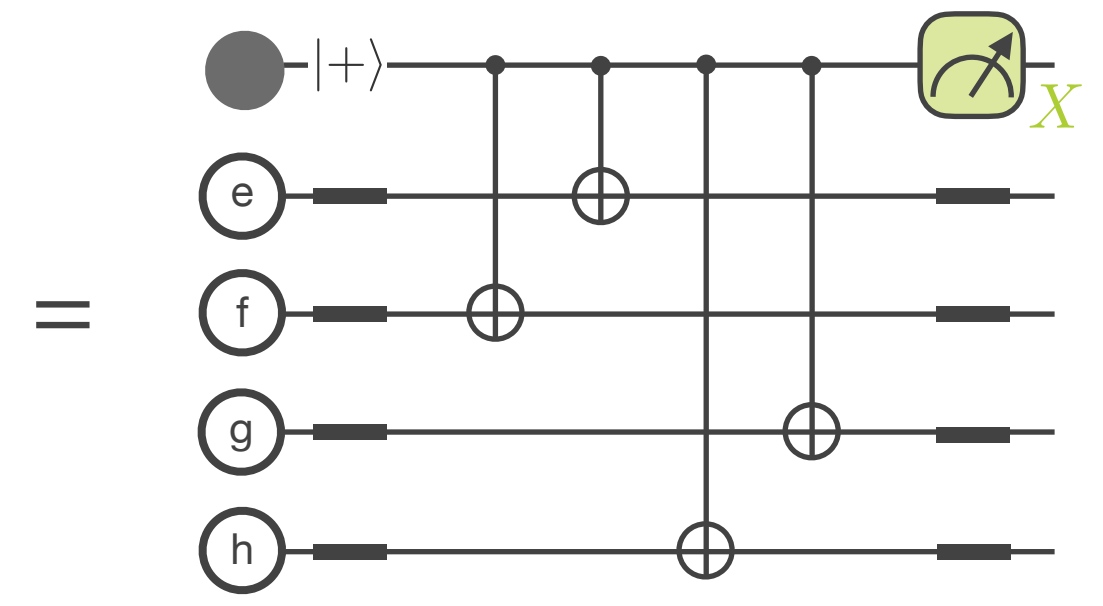
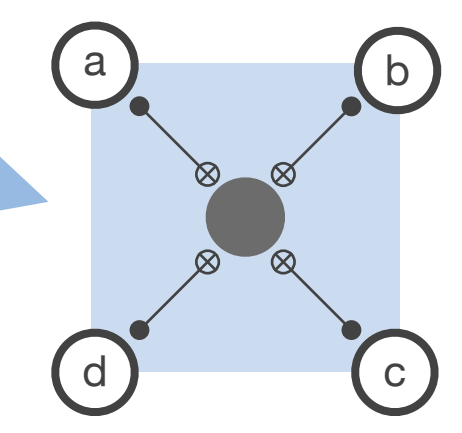
25 computational qubits
24 auxiliary qubits

d=5 code

X stabilizer



Z stabilizer

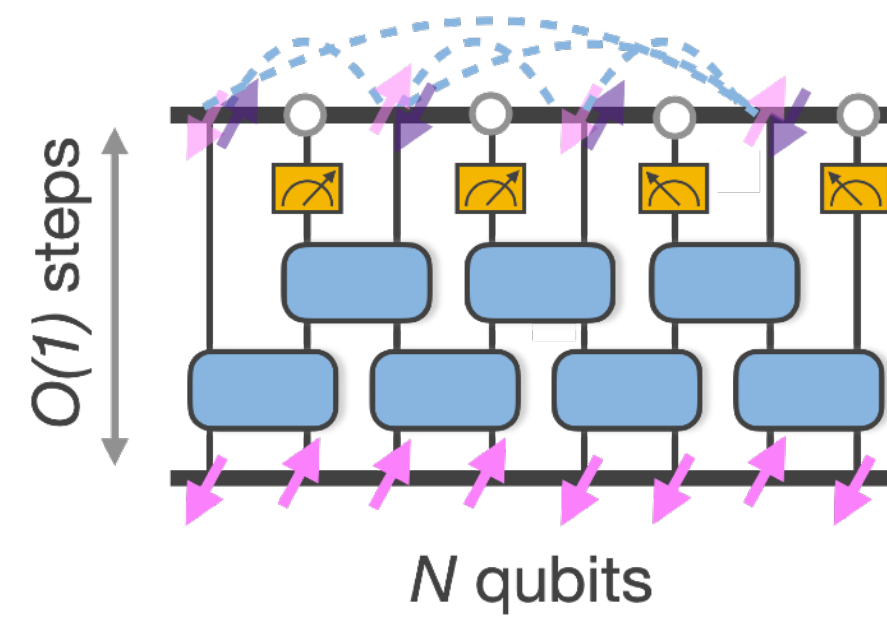


The toric/surface code was conceived as a **measurement protocol.**

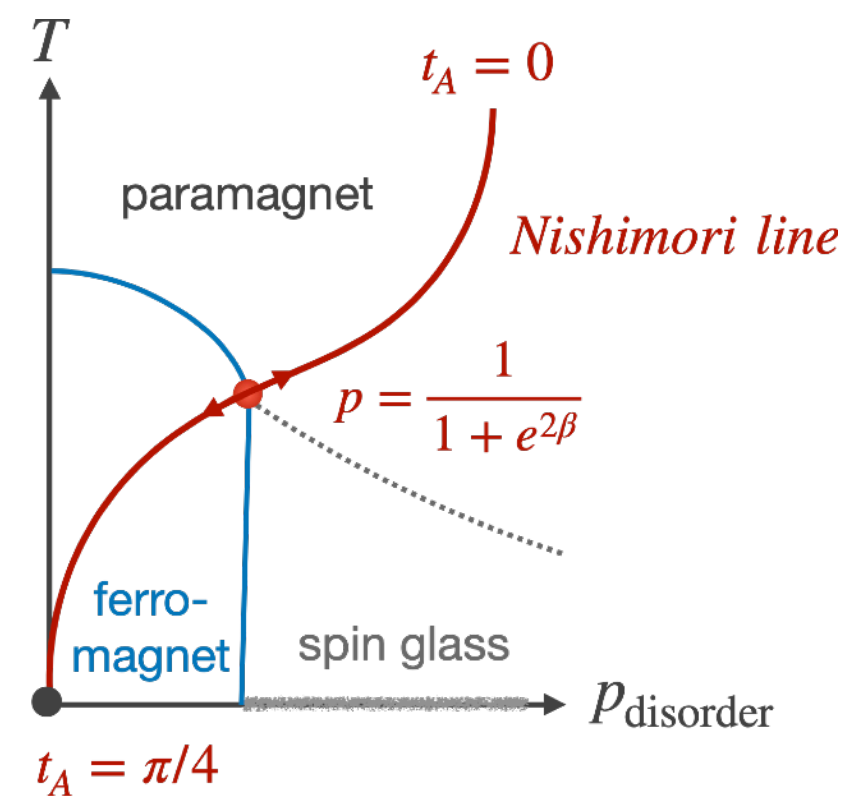
measurement & entanglement



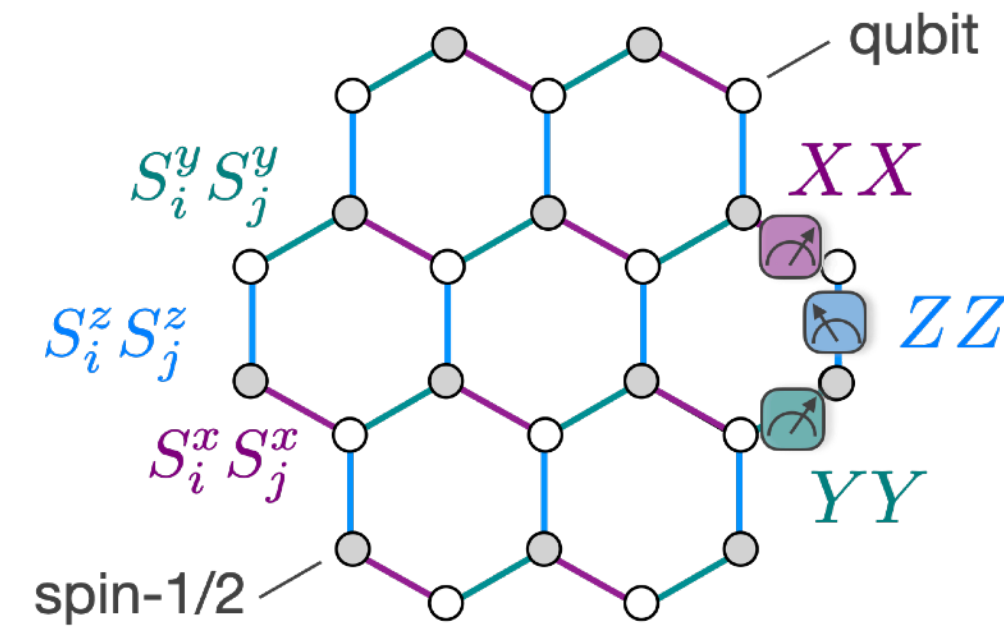
measurement-assisted state preparation



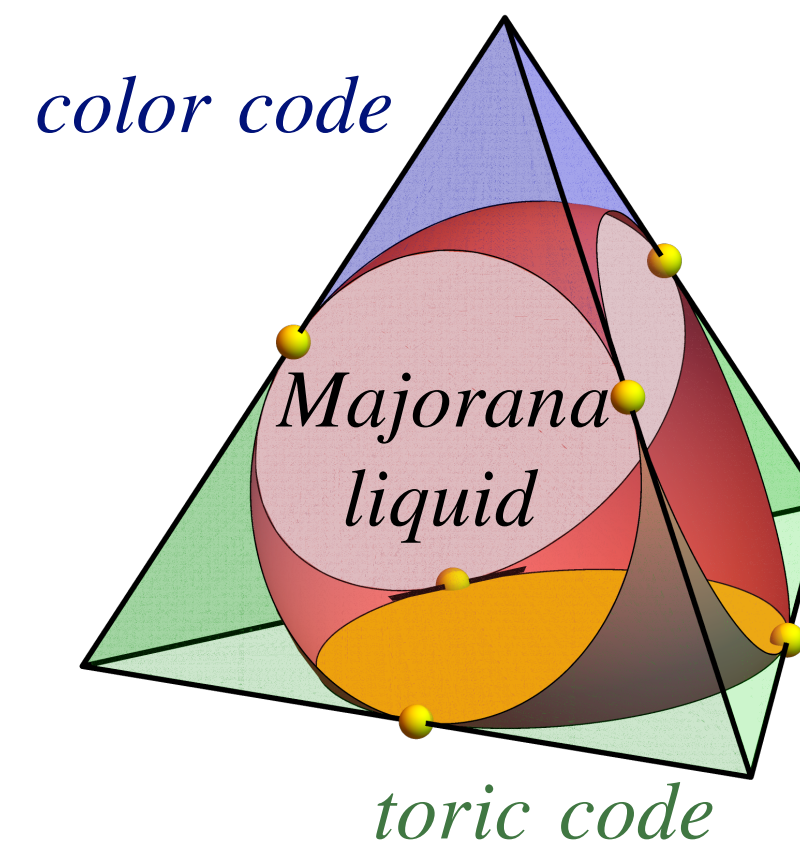
Nishimori's cat



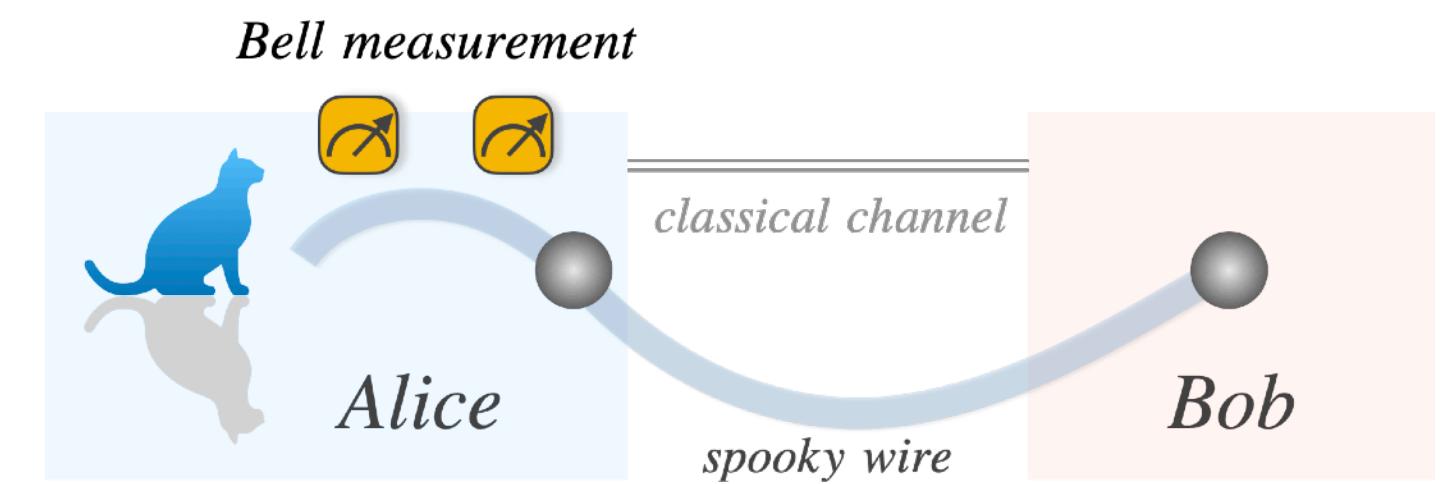
measurement-only quantum dynamics



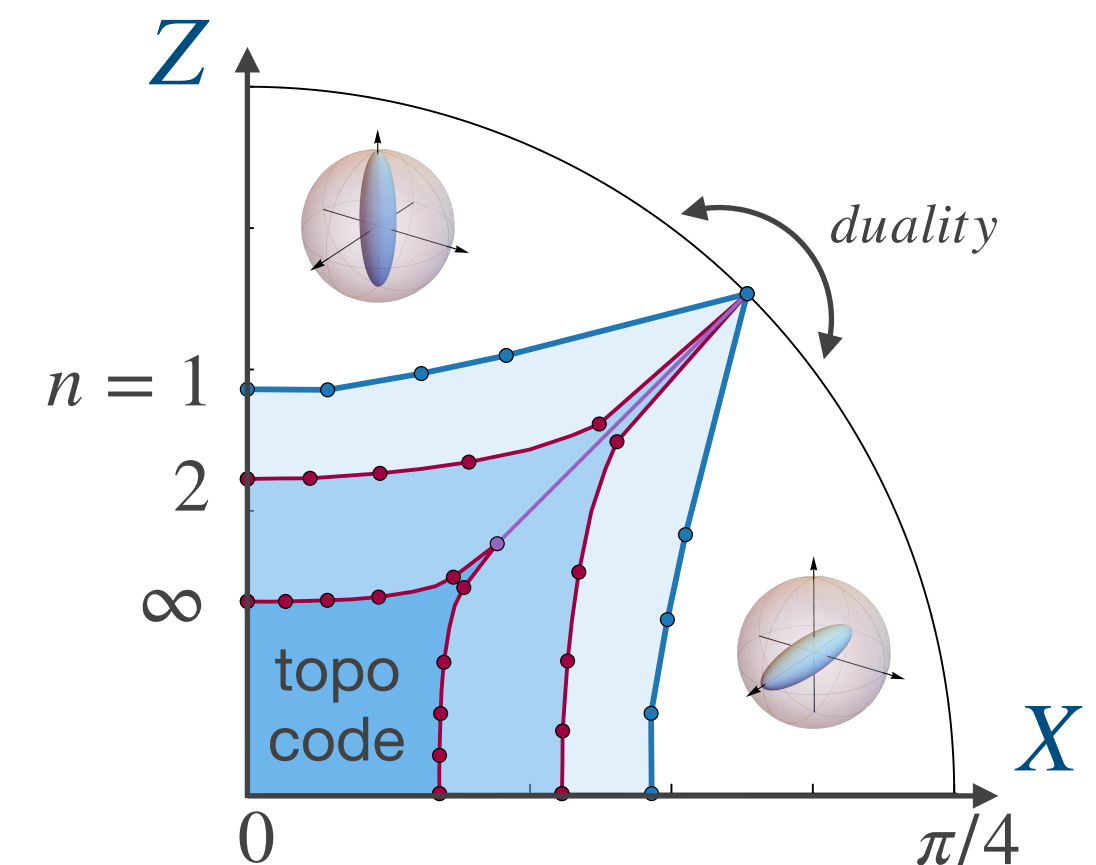
Kitaev circuits

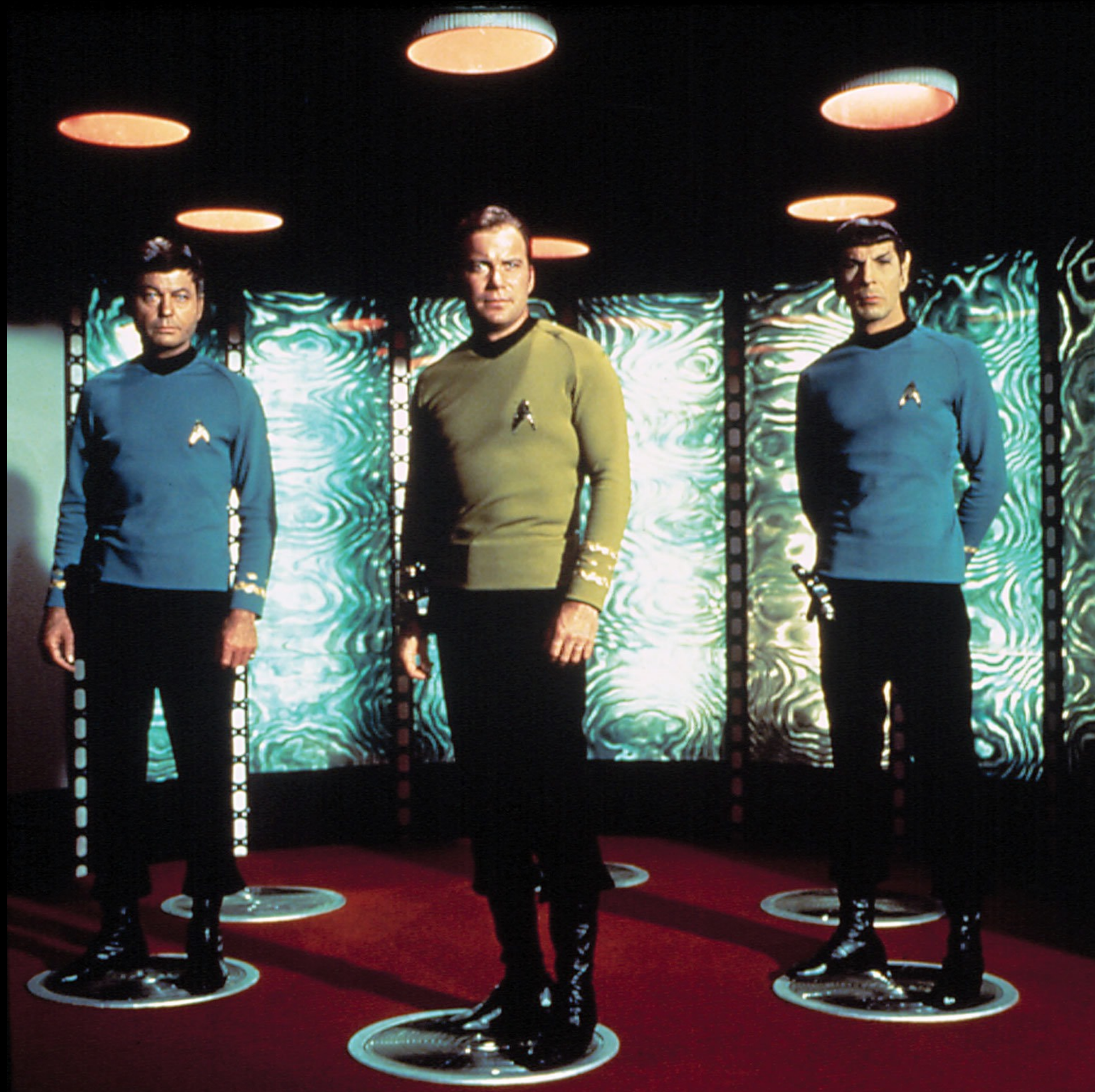


many-qubit teleportation



Bell decoders





teleportation

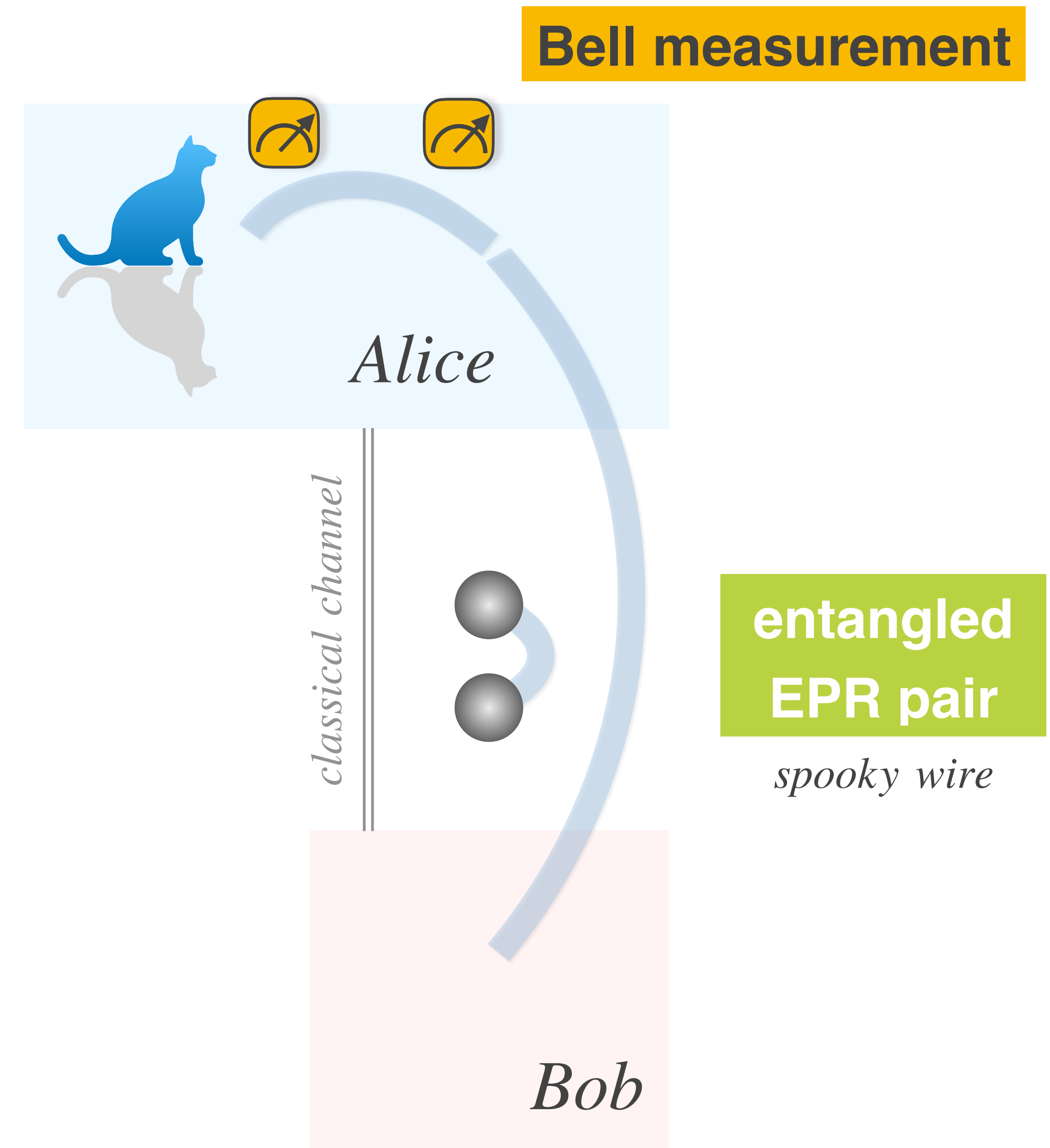
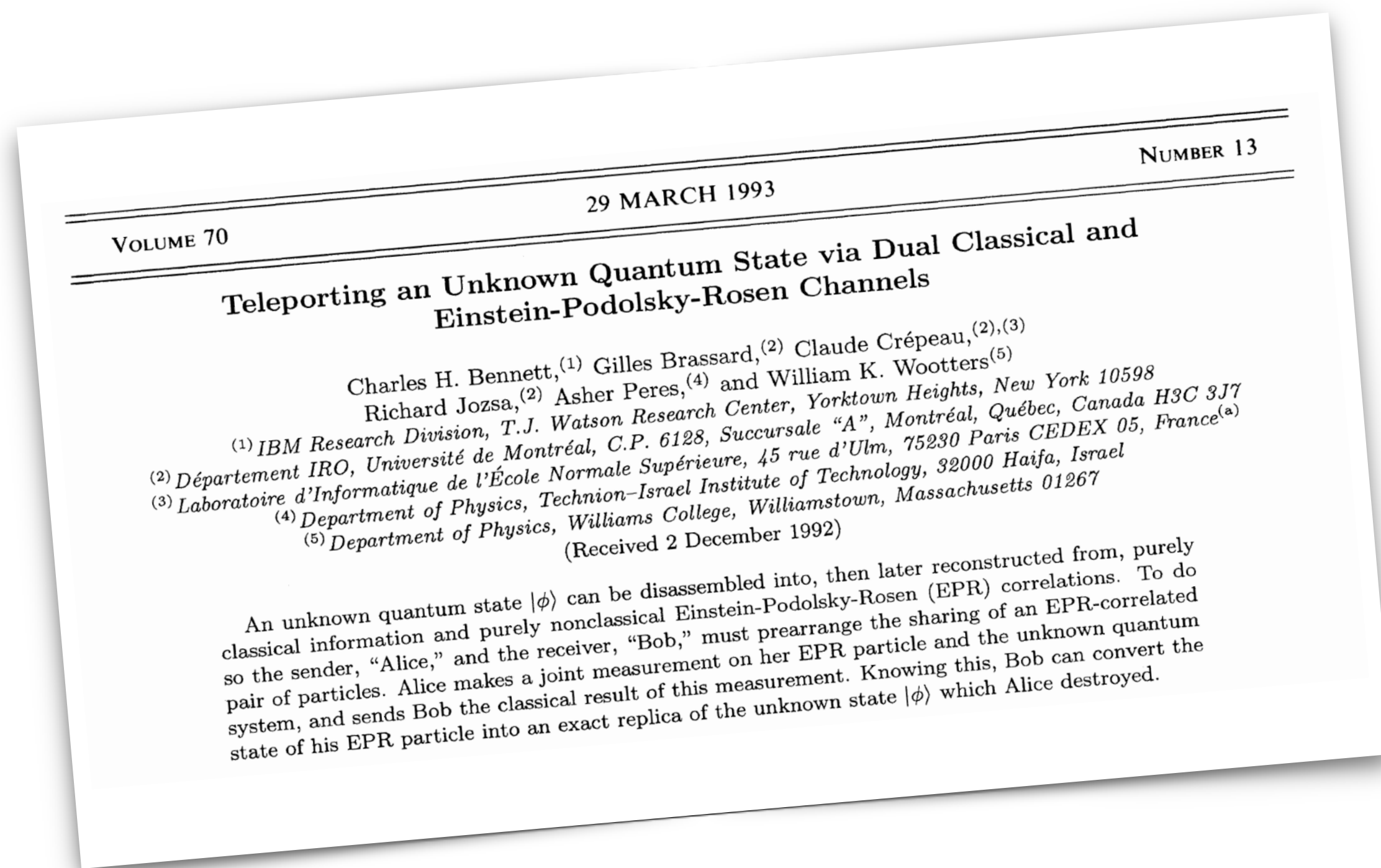
science meets **fiction**

quantum teleportation

teleportation?

quantum mechanics

to the rescue ...

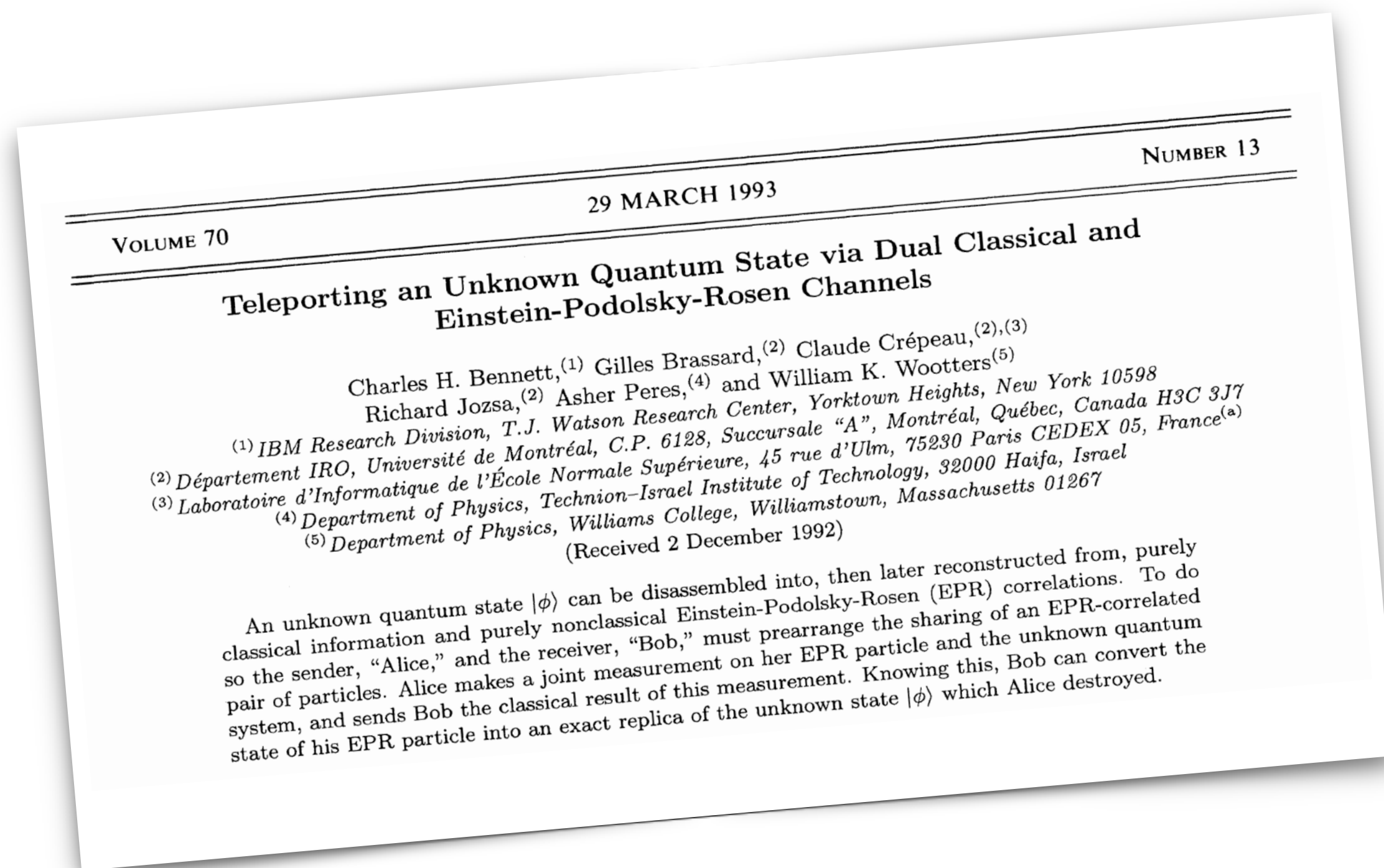


quantum teleportation

teleportation?

quantum mechanics

to the rescue ...



experimental quantum teleportation (1997)

D. Bouwmeester, ..., A. Zeilinger, *Nature* **390**, 575 (1997)



3m teleportation on demand (2014)

W. Pfaff, ..., R. Hanson, *Science* **345**, 532 (2014)

The New York Times

100km optical fibre channel (2016)

Q.-C. Sun *et al.*, *Nature Photonics* **10**, 671 (2016)



1000km earth-satellite channel (2017)

J.-G. Ren *et al.* *Nature* **549**, 70 (2017)

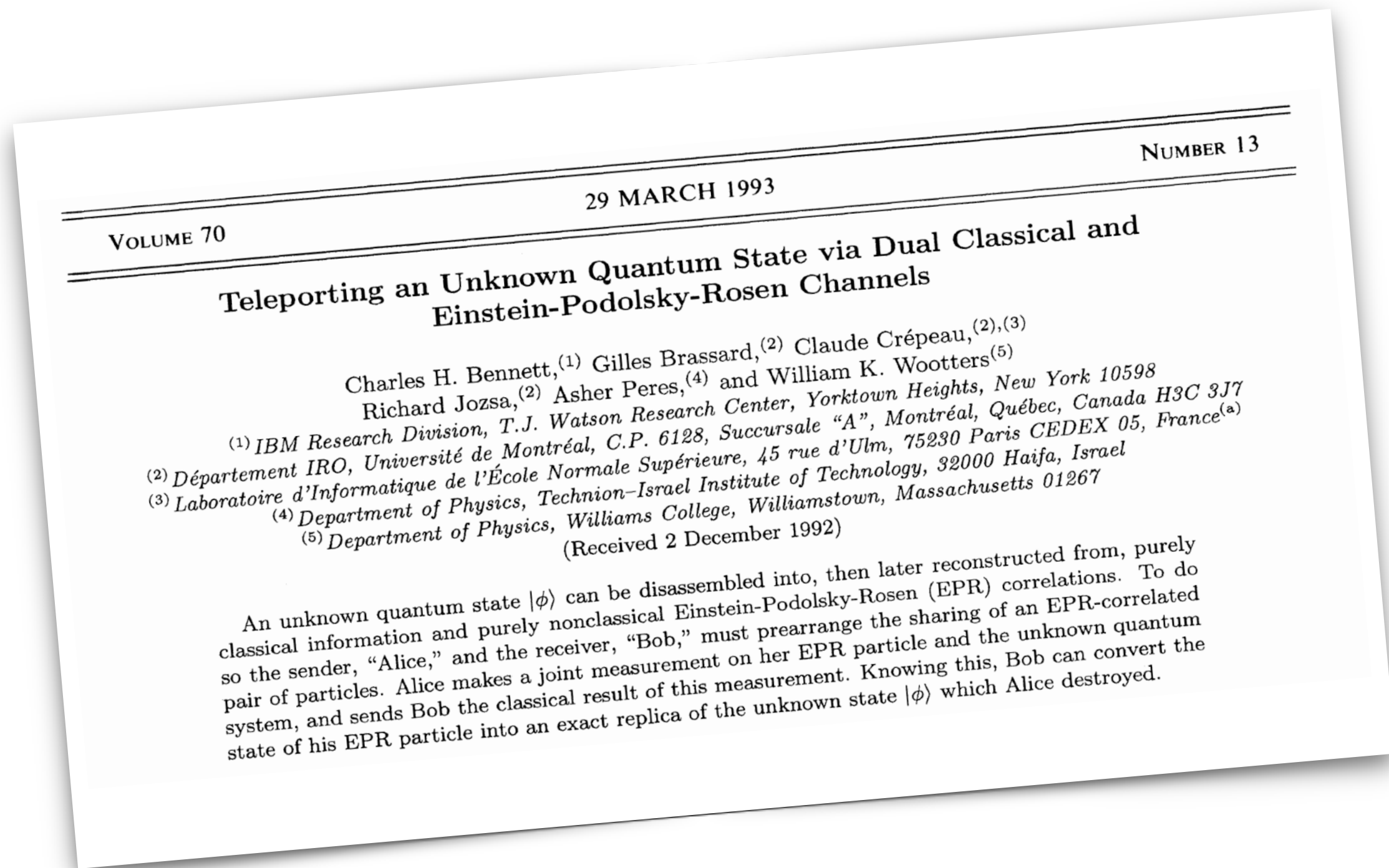


quantum teleportation

teleportation?

quantum mechanics

to the rescue ...

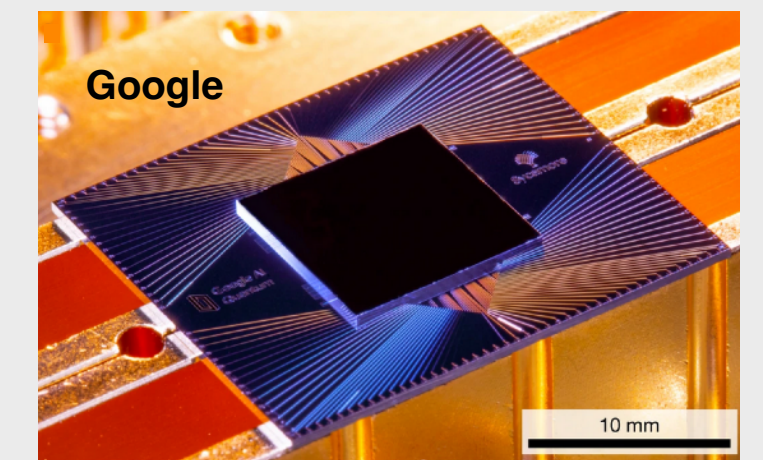


single qubit teleportation



few-qubit teleportation

noisy intermediate-scale
NISQ quantum devices

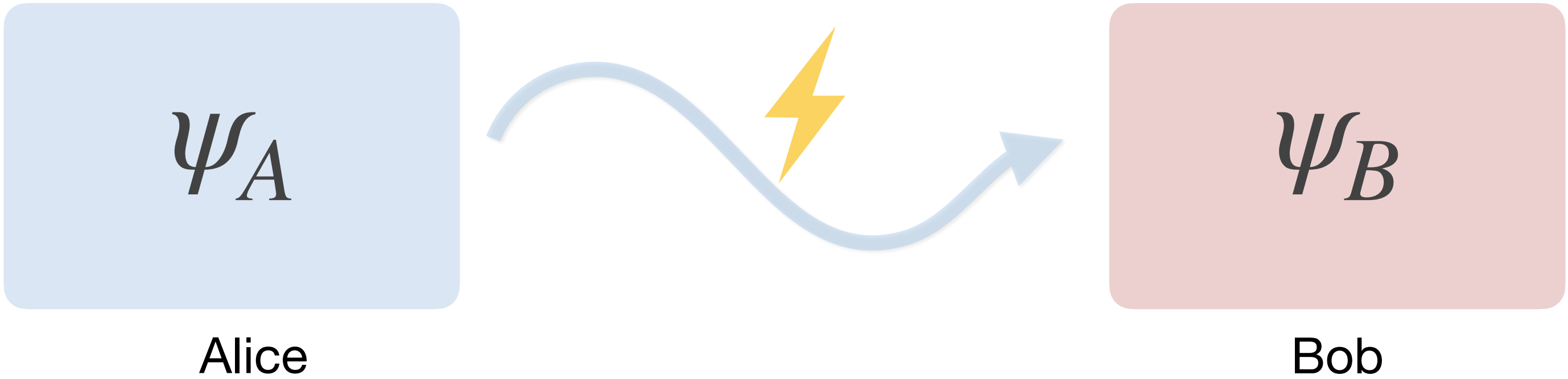


many-body teleportation

Star Trek

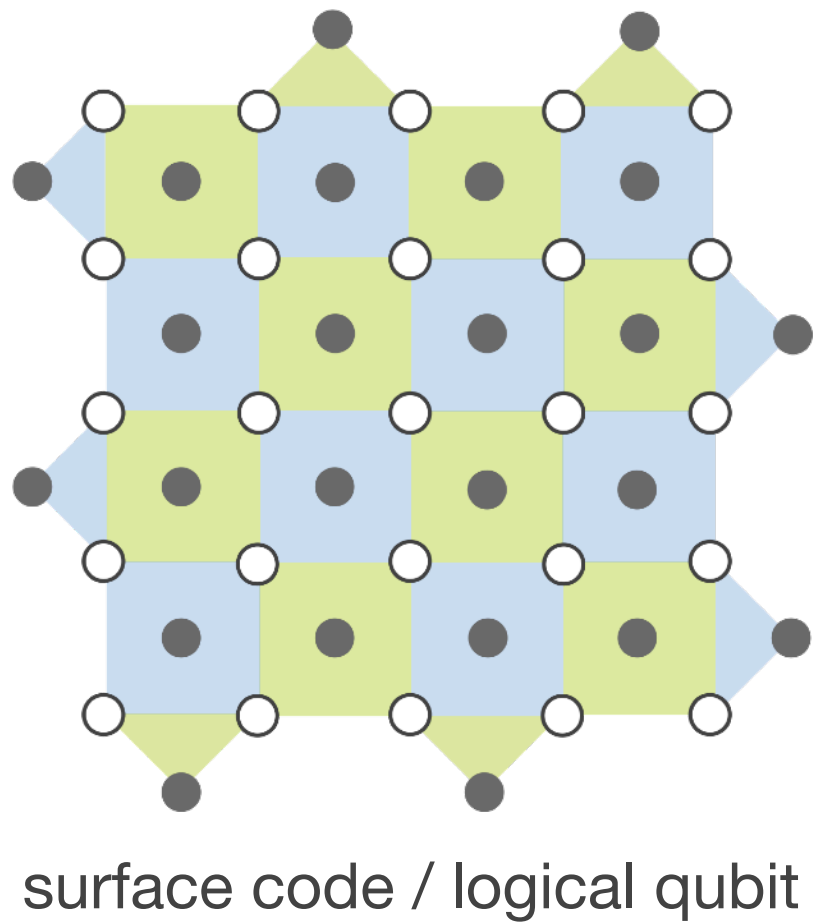
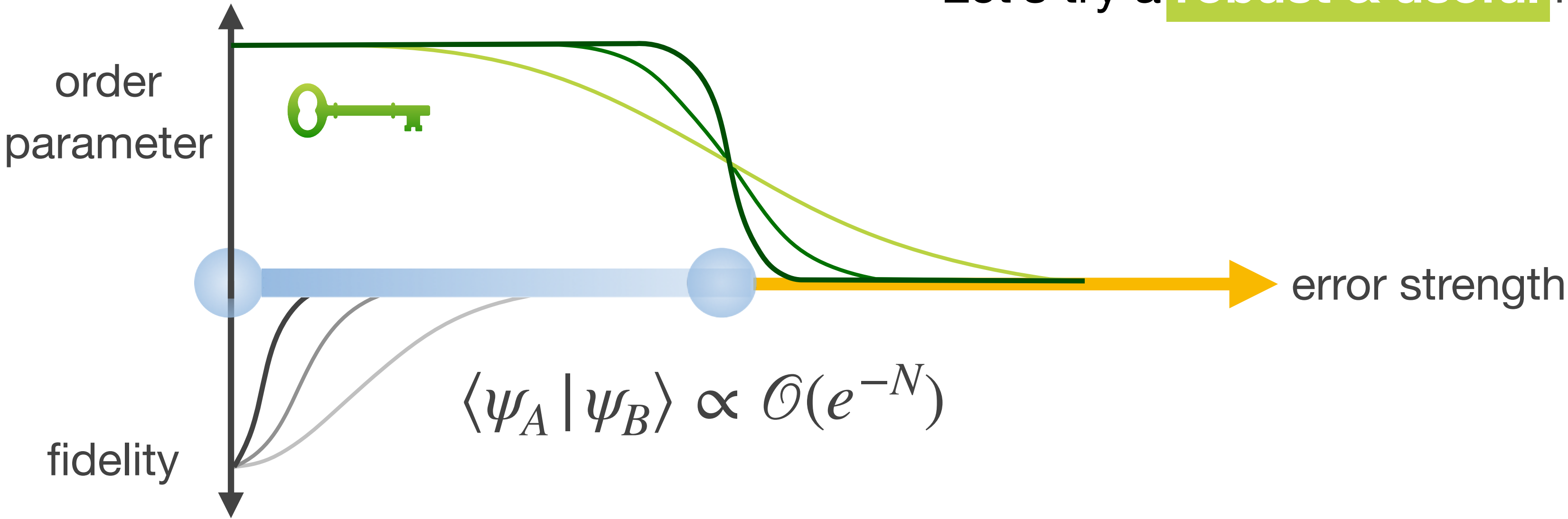


teleportation of quantum matter

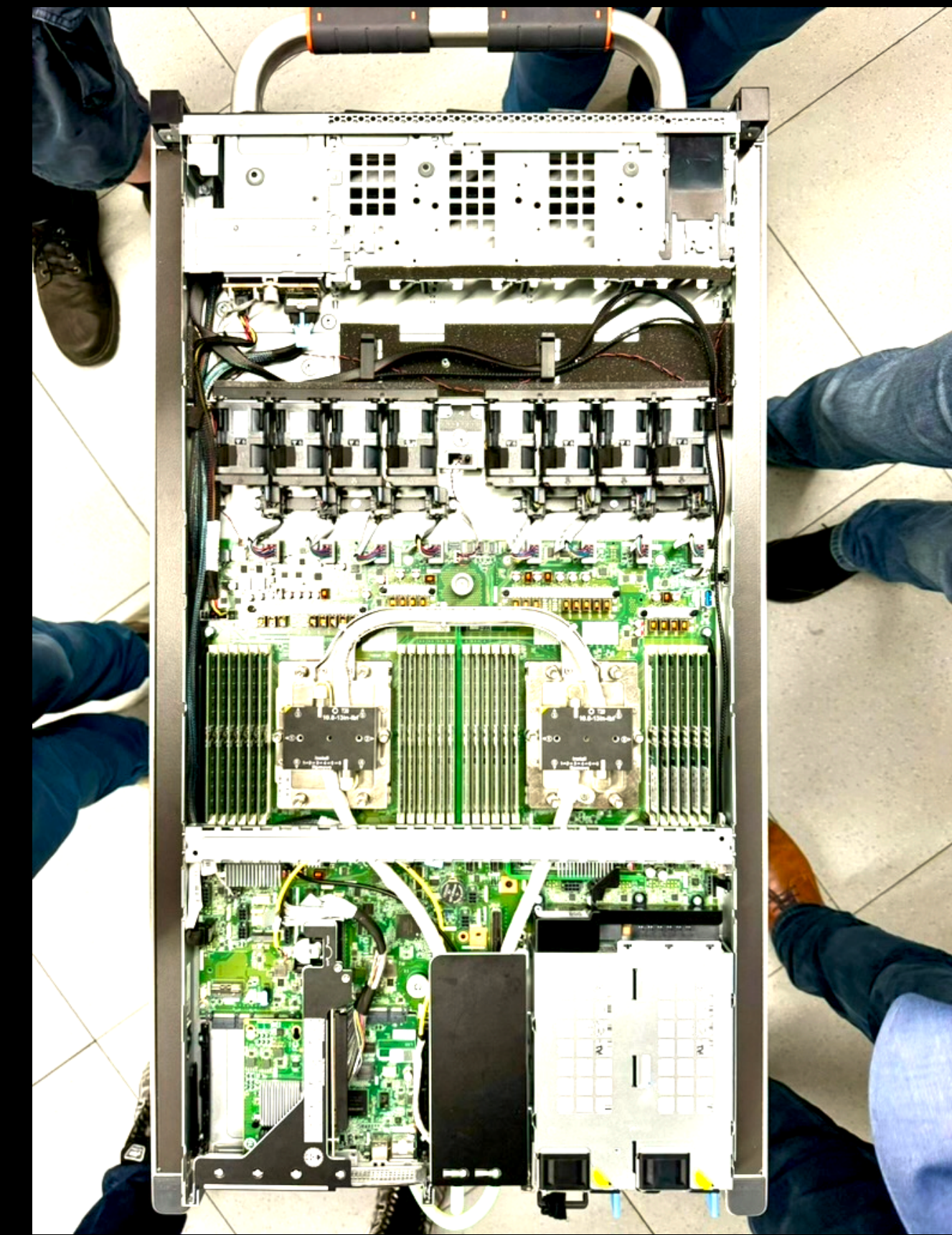
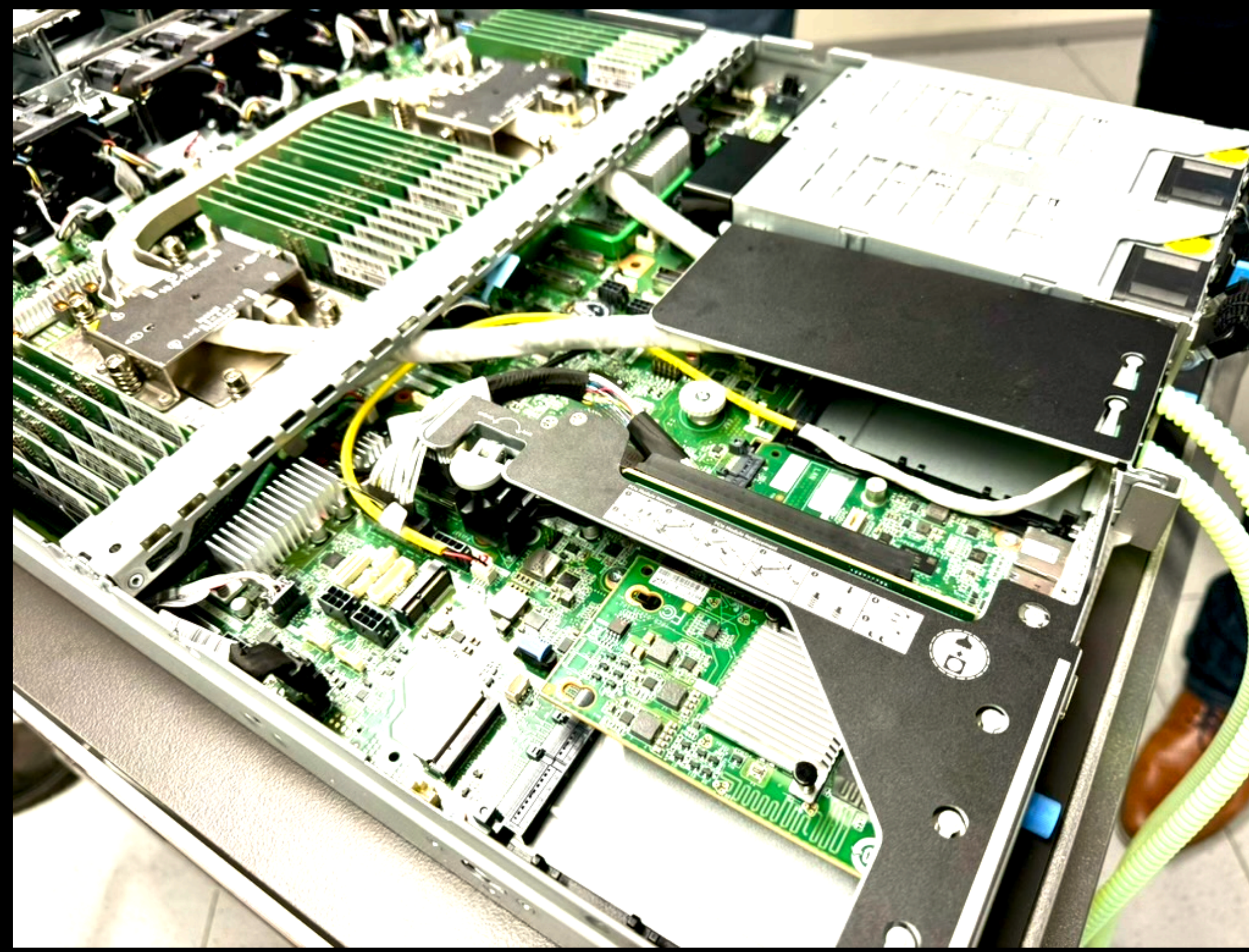
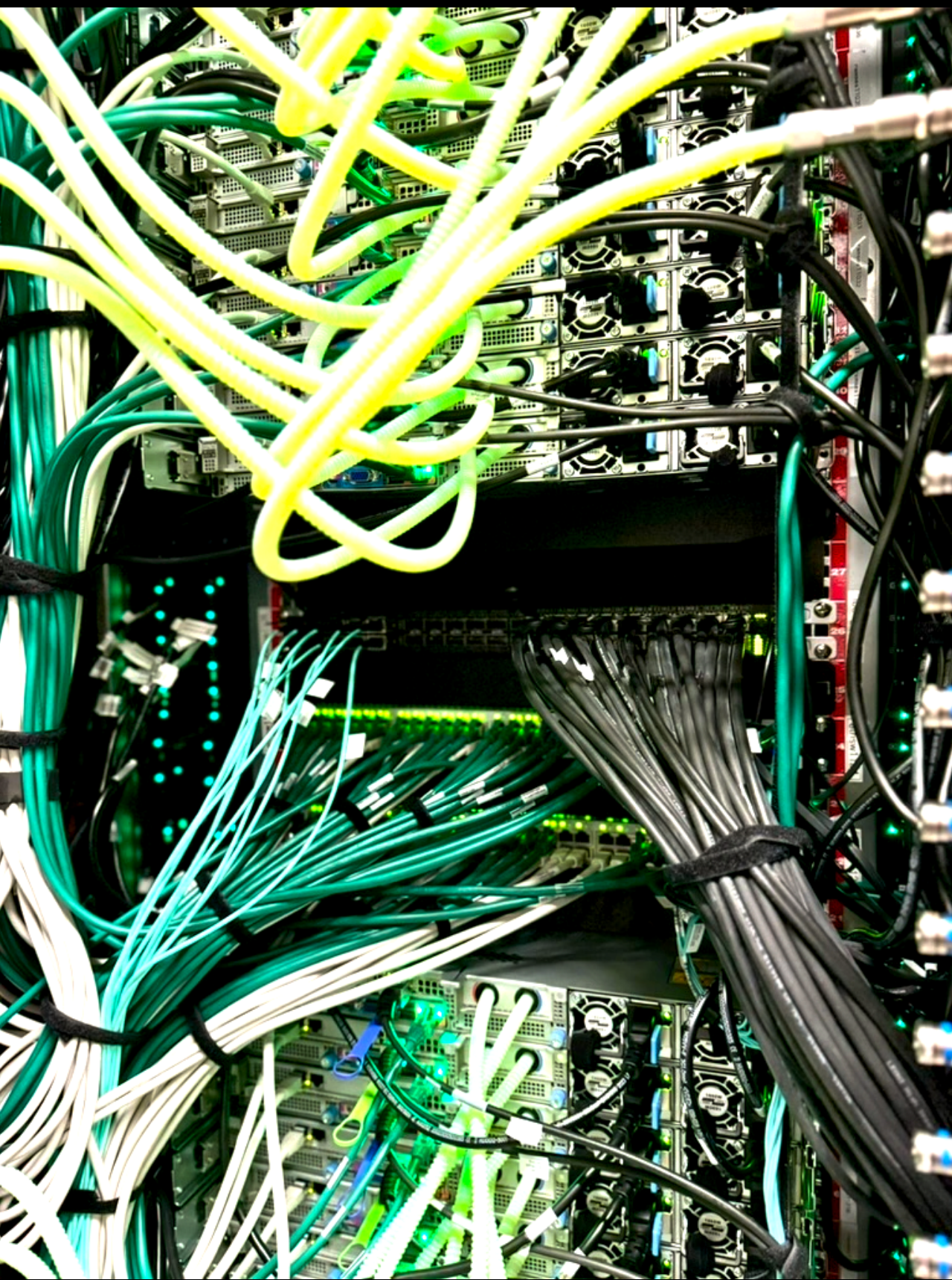


Q: Can quantum matter be teleported under coherent error?

Let's try a robust & useful many-body state



von Neumann architecture



RAMSES HPC cluster @ University of Cologne, September 2024

the team



Finn Eckstein

University of Cologne



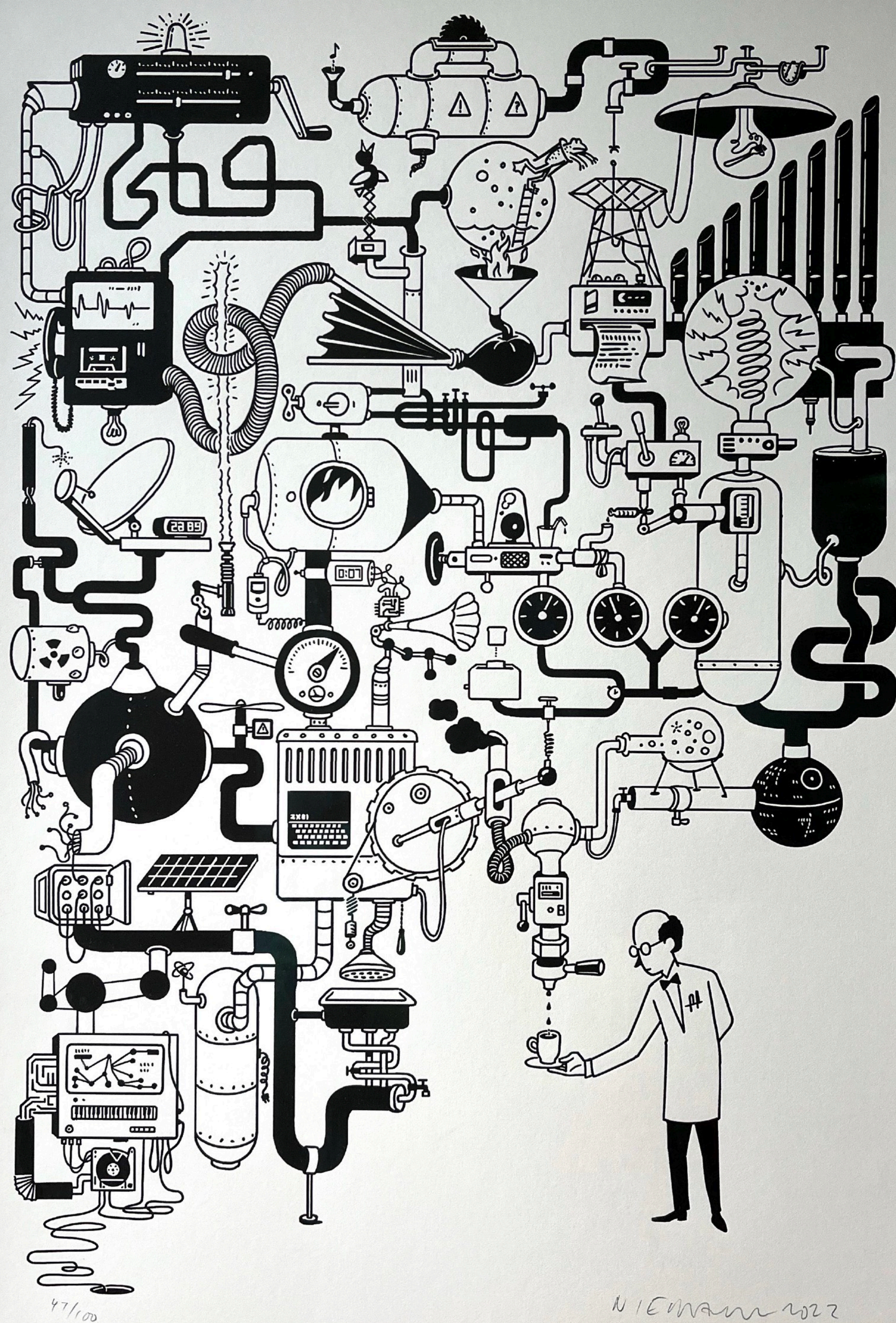
Guo-Yi Zhu

University of Cologne
Hong Kong University of Science and Technology



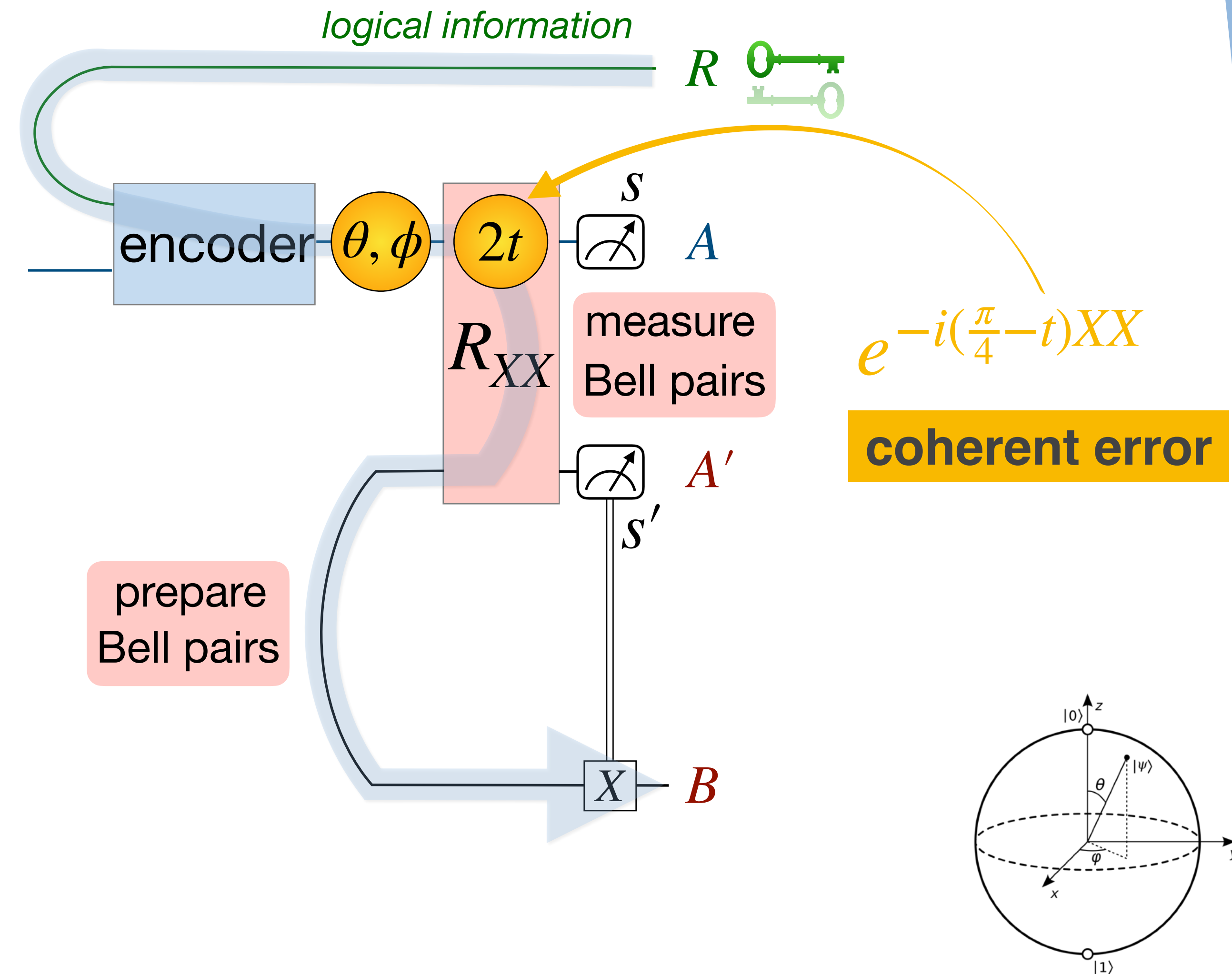
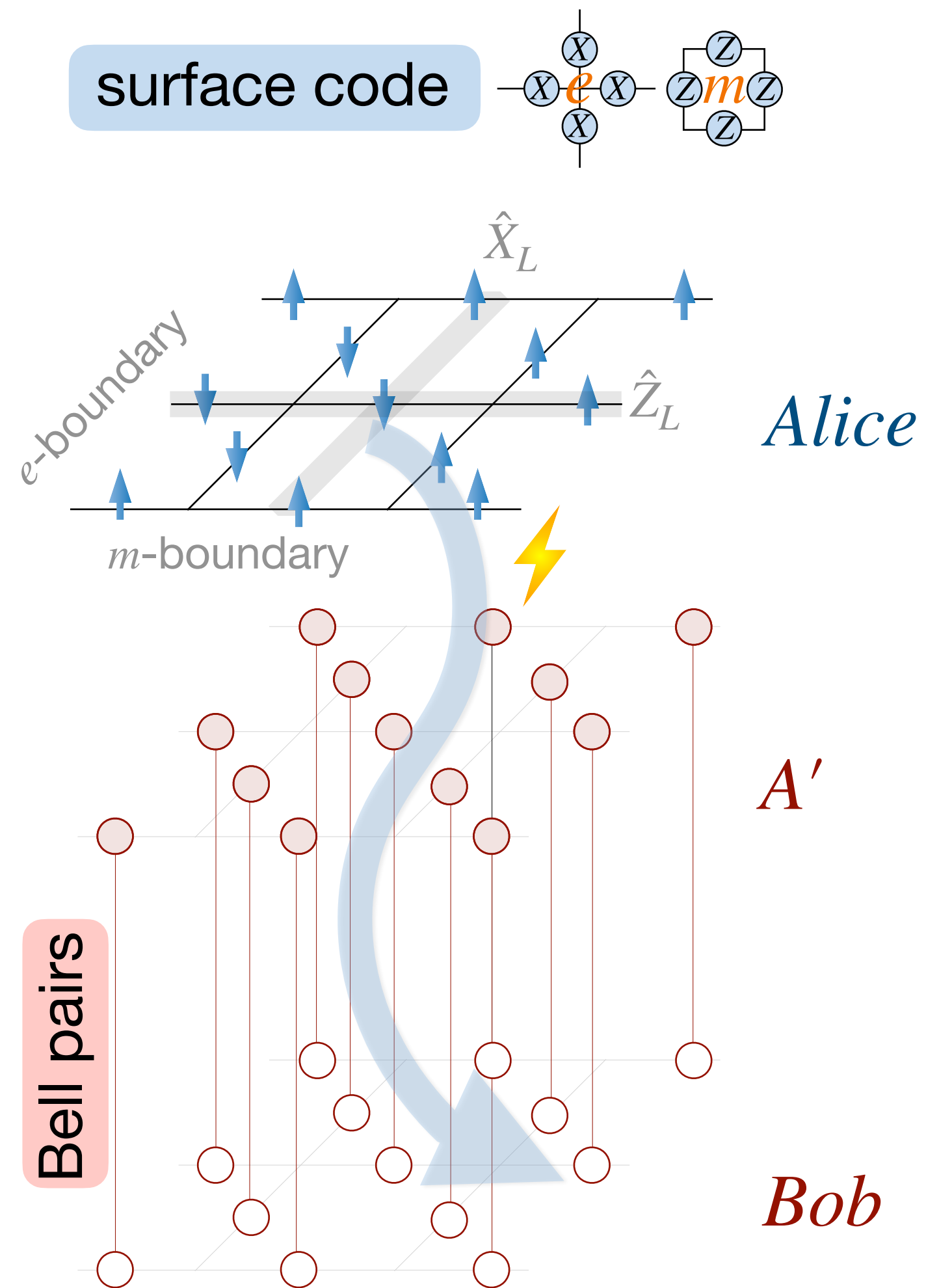
Bo Han

Weizmann Institute of Science

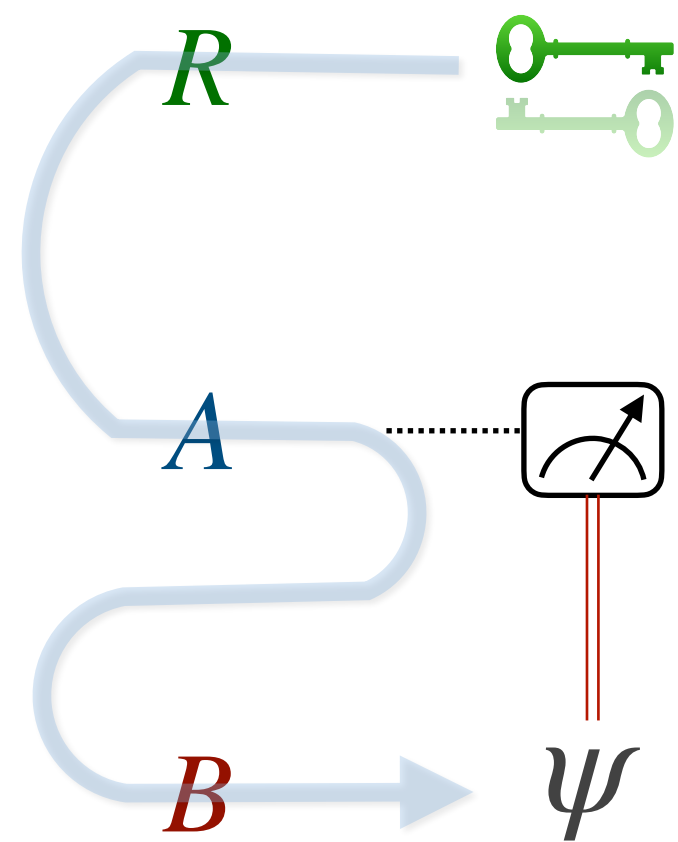


teleportation protocol

protocol & quantum circuit



diagnose robust teleportation



$$I_c = S_{RA} - S_A$$

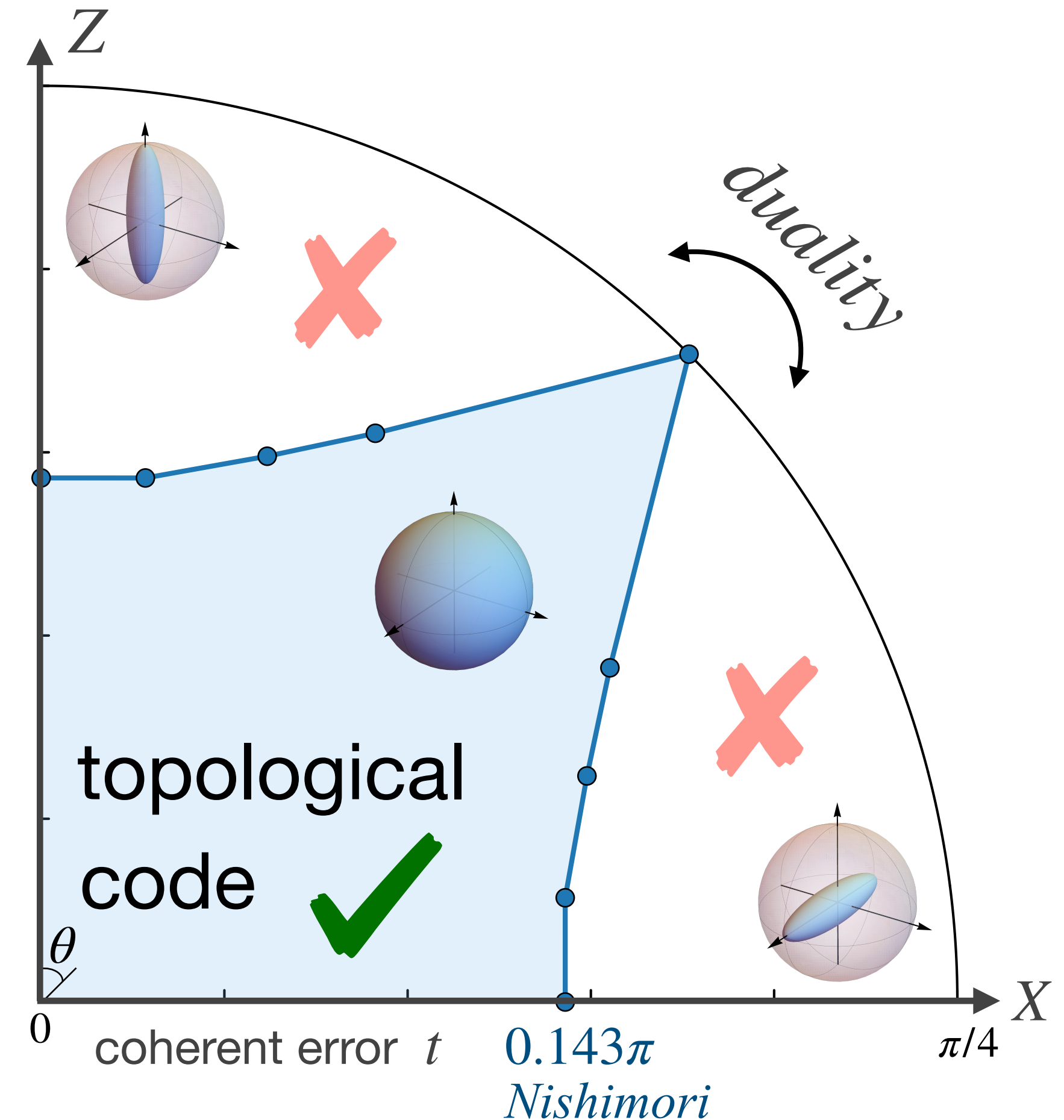
$$= S_{AB} - S_{RAB}$$

$$= \sum_{\mathbf{s}} P(\mathbf{s}) S_B(\mathbf{s})$$

coherent information

channel capacity

- Can **Alice steal** the key?
- Can **Bob decode** the key with classic info shared by Alice?
- **Ensemble average** of logical entropy (size of code space).

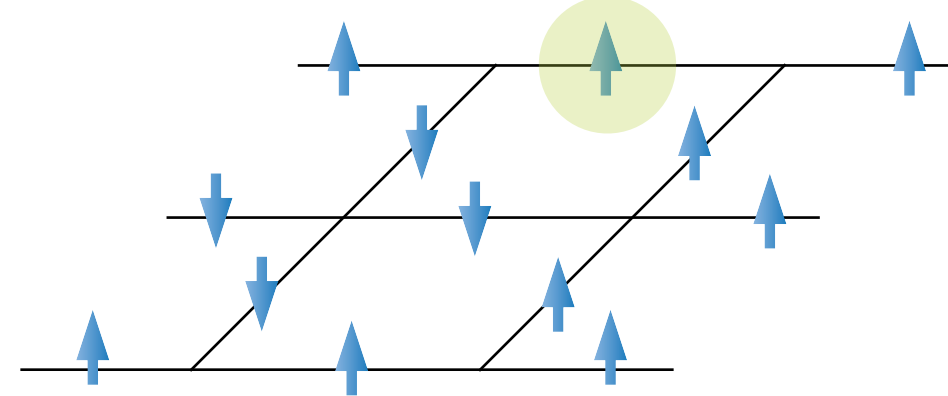
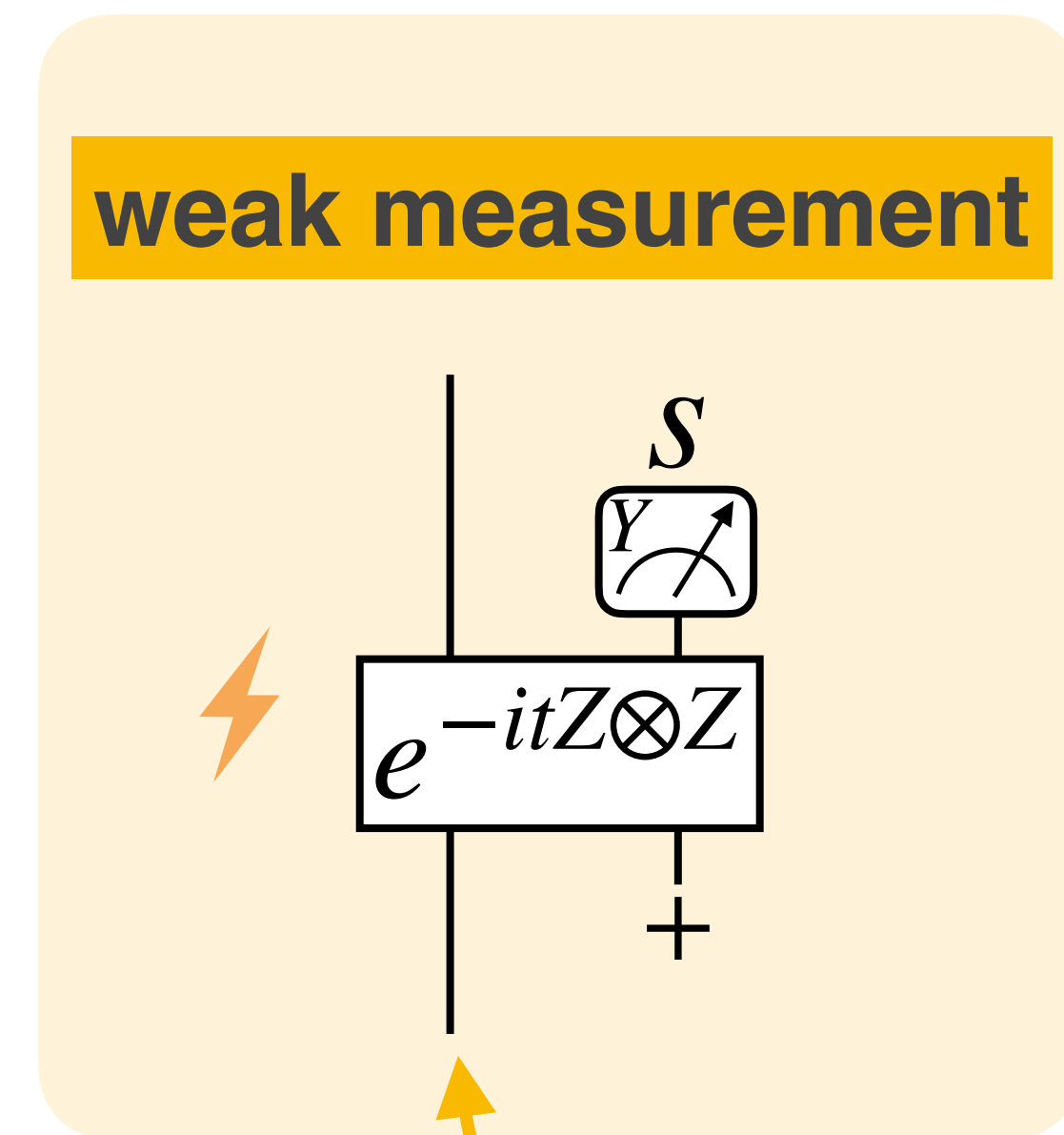
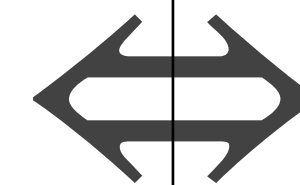
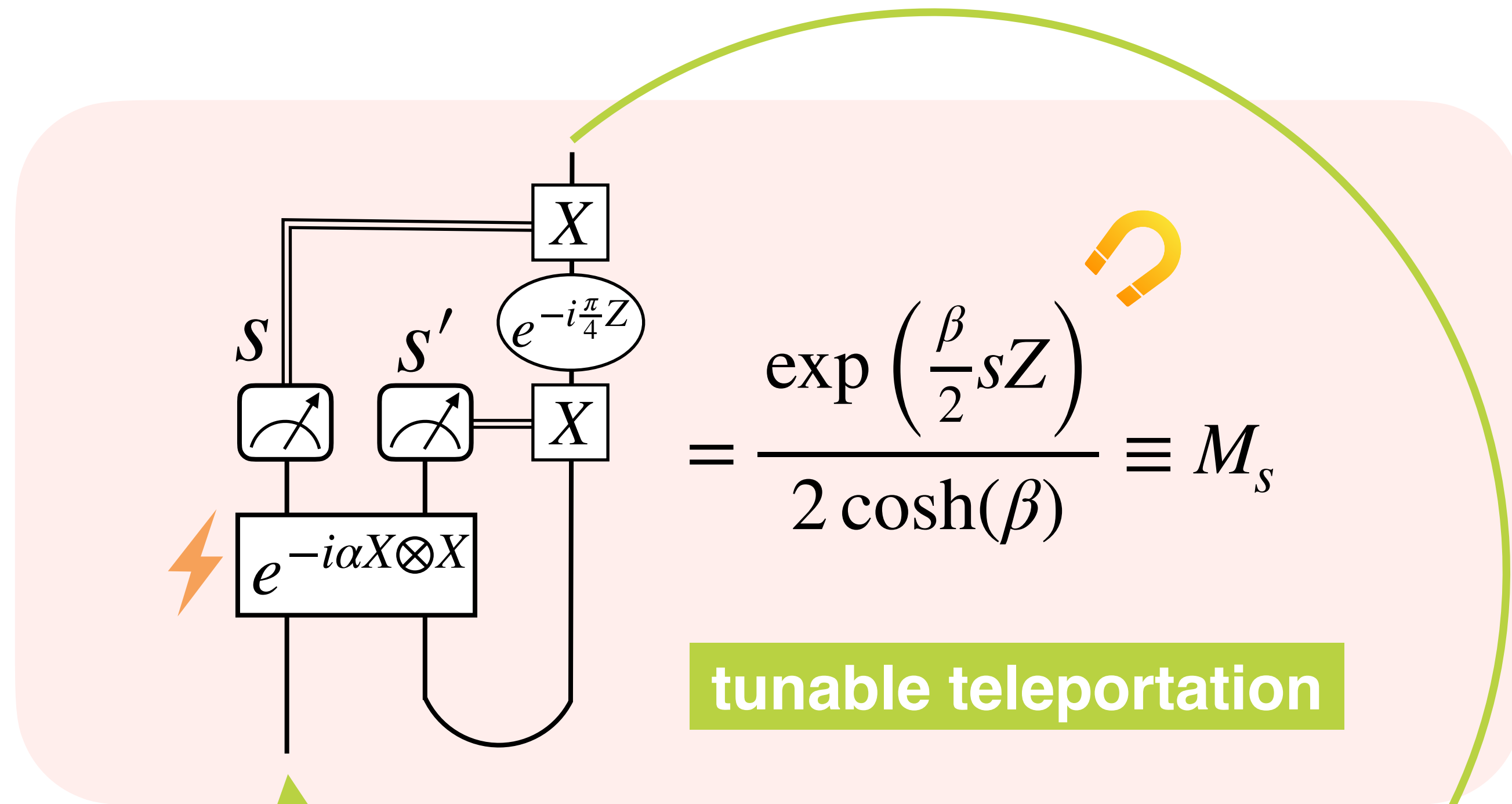


$$\rho_{RAB} = \sum_{\mathbf{s}} P(\mathbf{s}) |\Psi(\mathbf{s})\rangle_{RB} \langle \Psi(\mathbf{s})| \otimes |\mathbf{s}\rangle_A \langle \mathbf{s}|$$

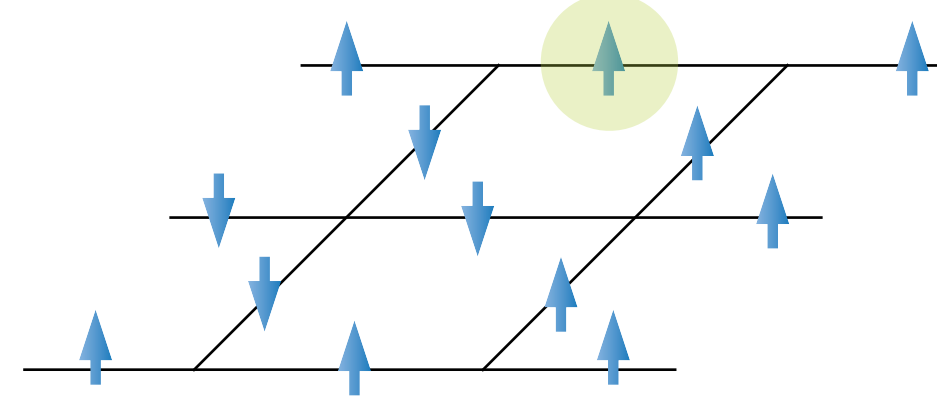
Schumacher, Nielsen 1996; Lloyd 1997; Gullans, Huse 2020;

Fan, Bao, Vishwanath, Altman 2023; Colmenarez, Huang, Diehl, Müller 2023

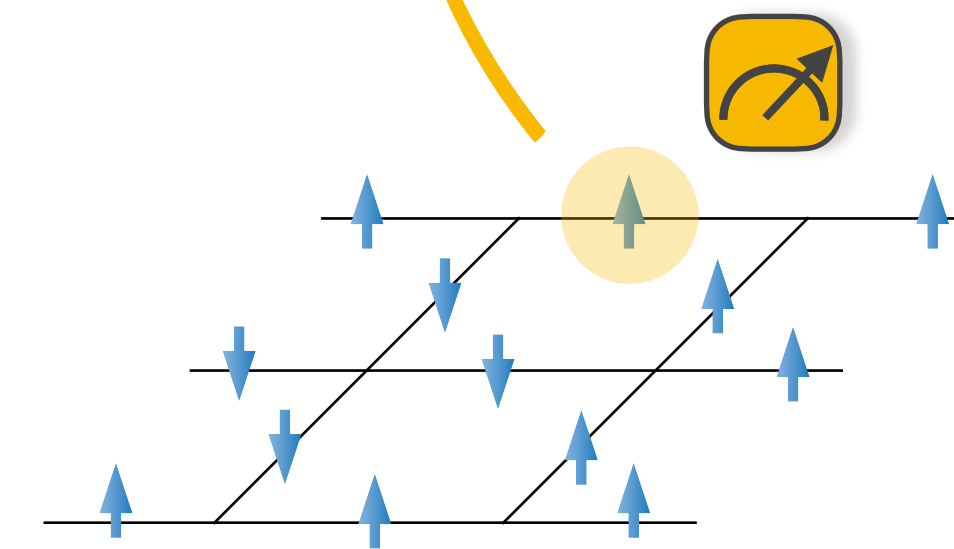
physical qubits / teleportation vs. measurement



Alice



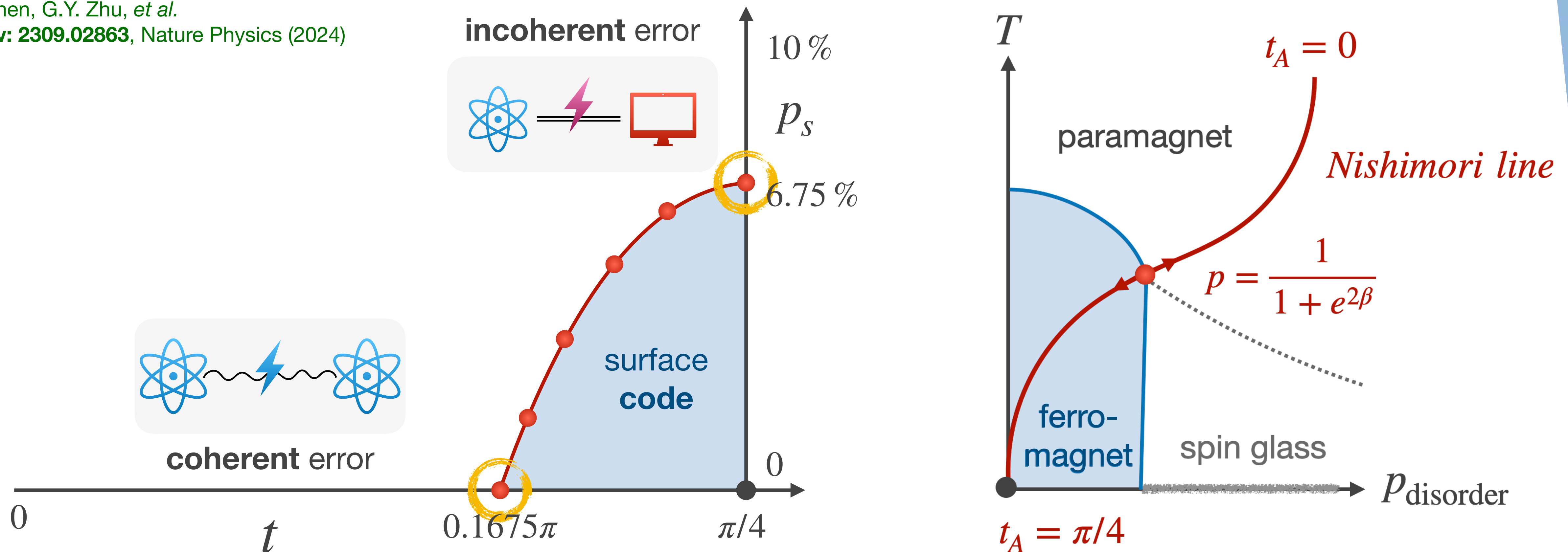
Bob



coherent vs. incoherent errors



Dennis, Kitaev, Landahl, Preskill 2002
 E. Chen, G.Y. Zhu, *et al.*
 arXiv: 2309.02863, Nature Physics (2024)



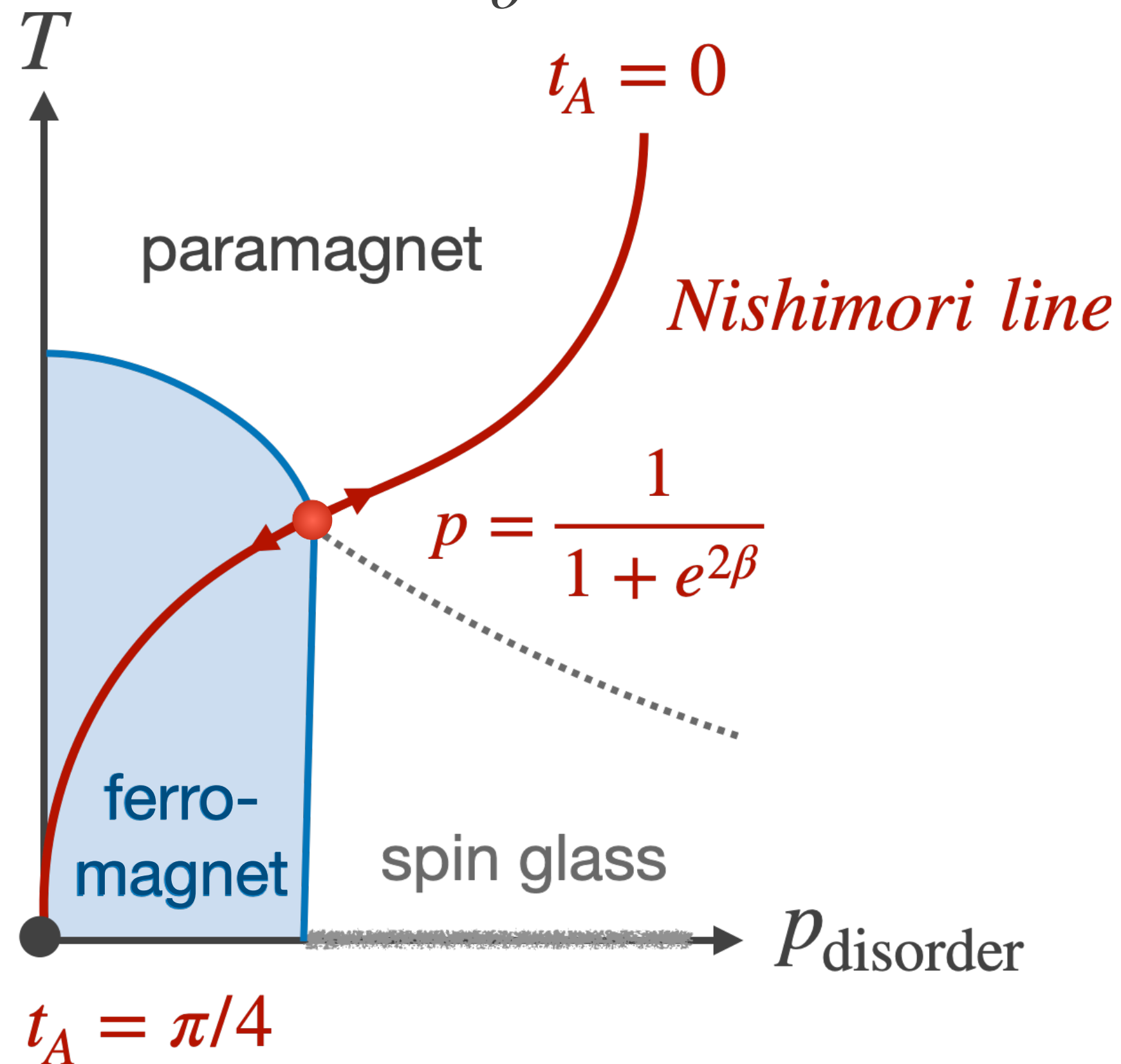
entire phase diagram is mapped to **Nishimori line**

$$\tilde{p} = \frac{1 - (1 - 2p_s)\sin(2t_A)}{2}$$

Nishimori criticality

map to random bond Ising model

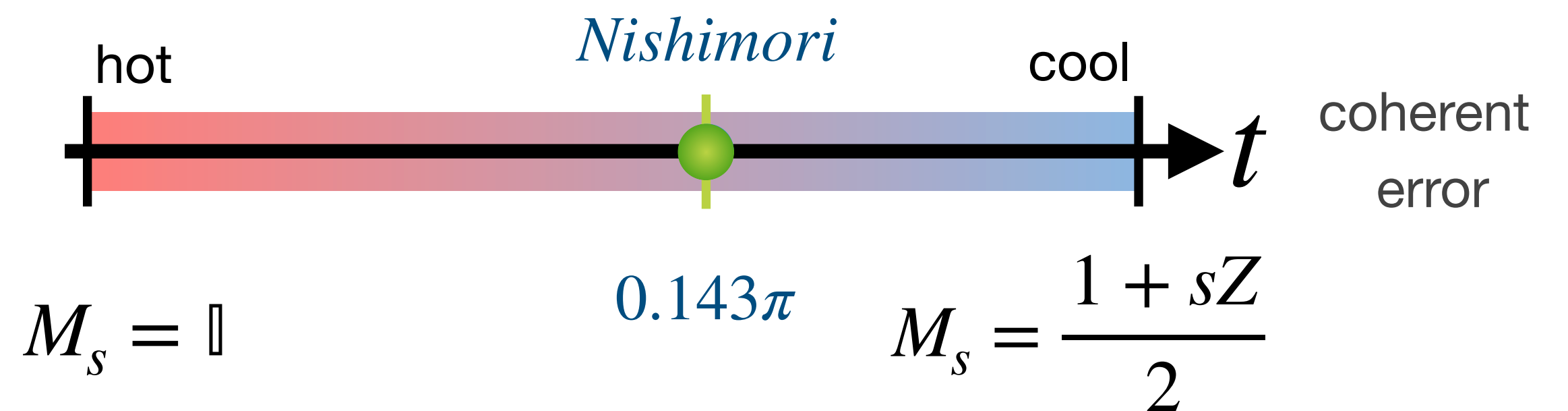
$$P(s) = \sum_{\sigma} e^{-\beta \sum_{ij} s_{ij} \sigma_i \sigma_j}$$



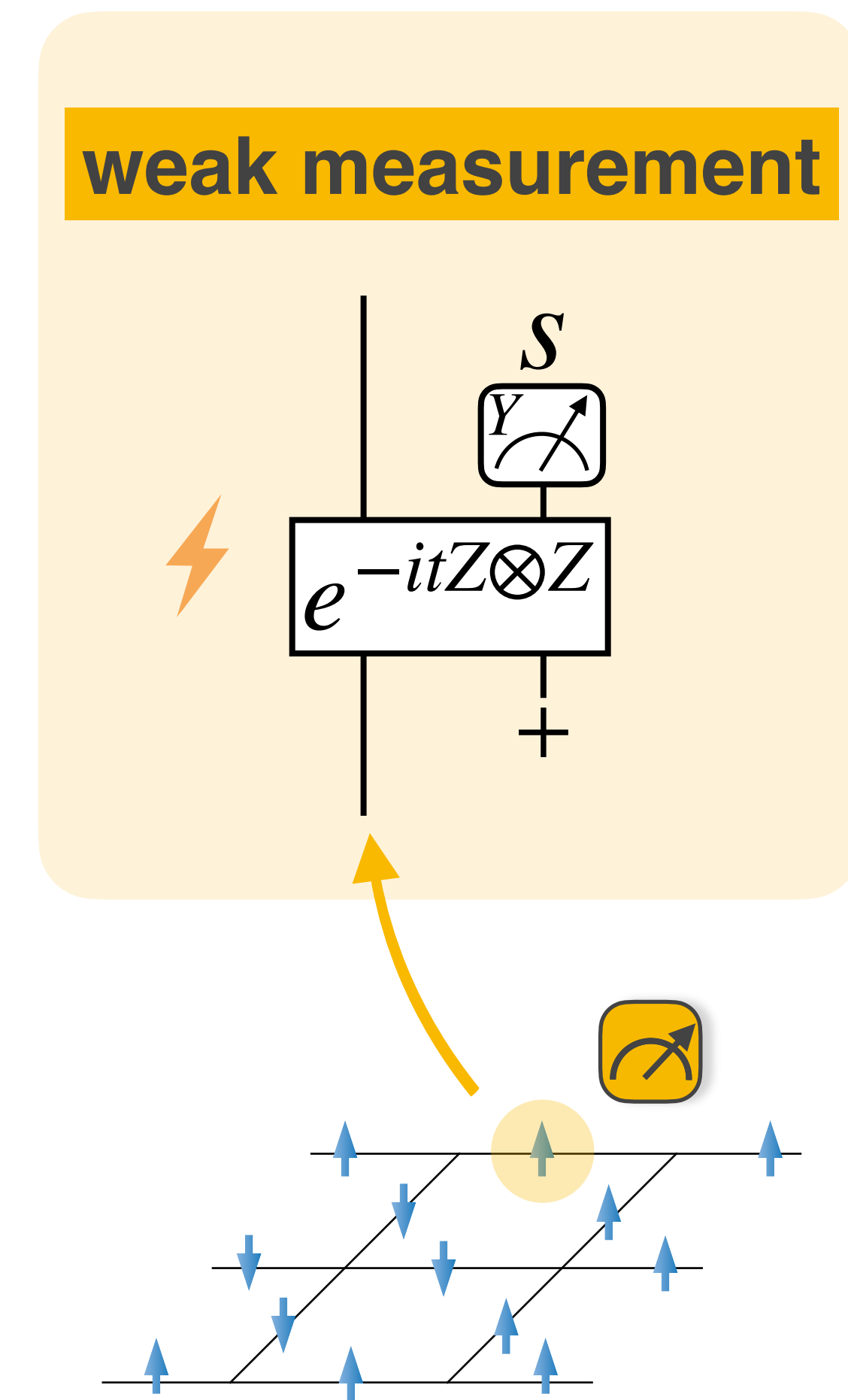
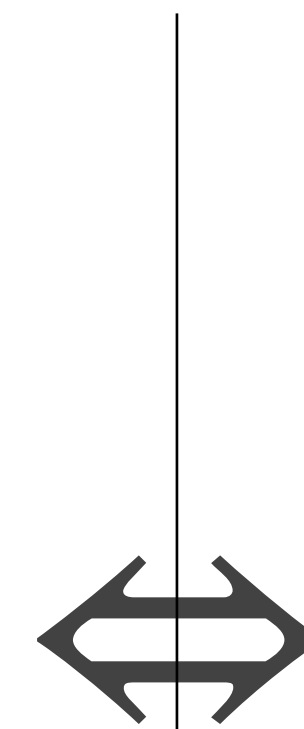
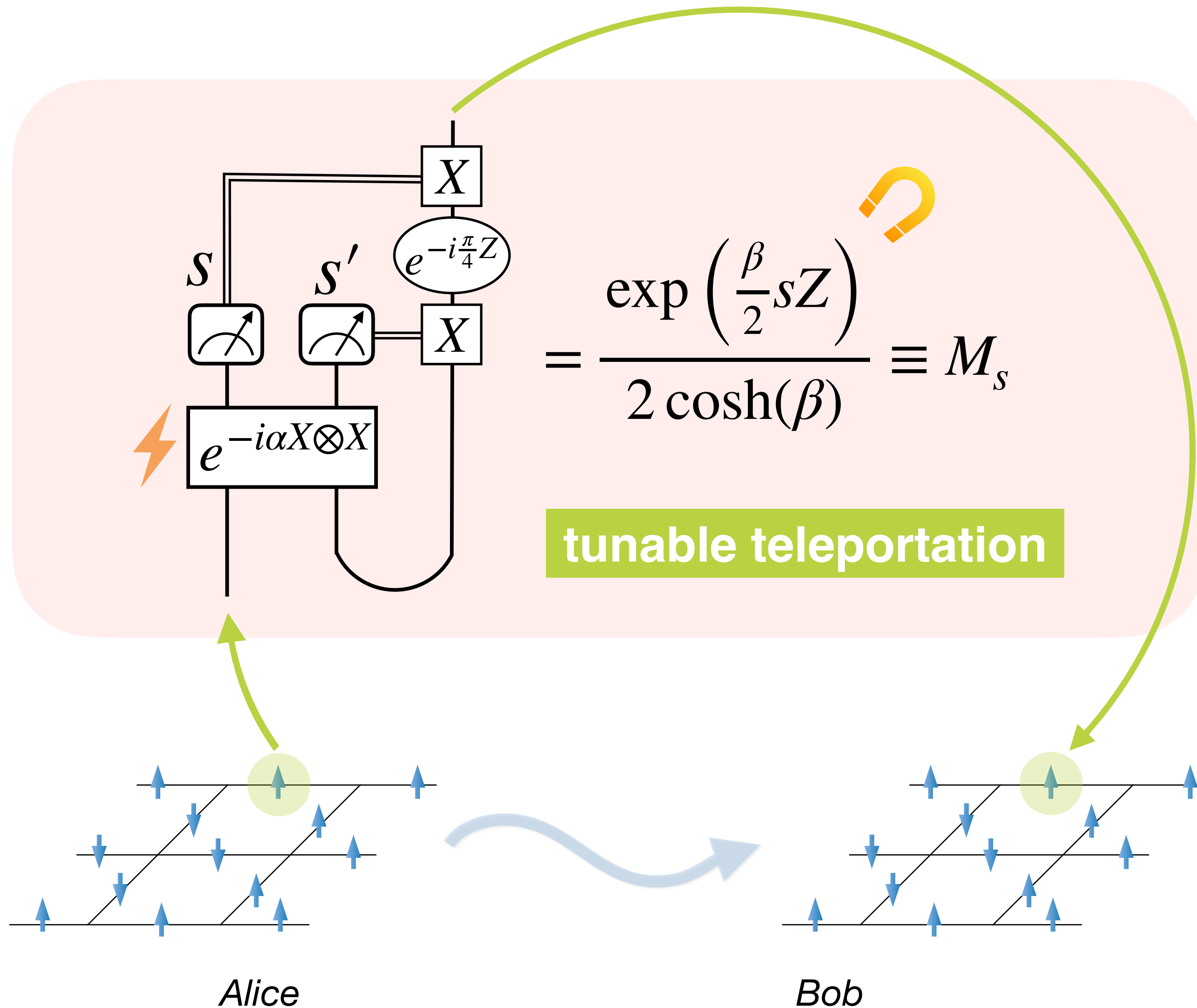
$$\rho = \sum_s \left[\begin{array}{c} \text{Alice} \\ |s\rangle\langle s| \end{array} \otimes \begin{array}{c} \text{Bob} \\ M_s |\Psi\rangle\langle\Psi| M_s \end{array} \right] \quad \theta = 0$$

$$M_s = e^{\frac{\beta}{2} s Z}$$

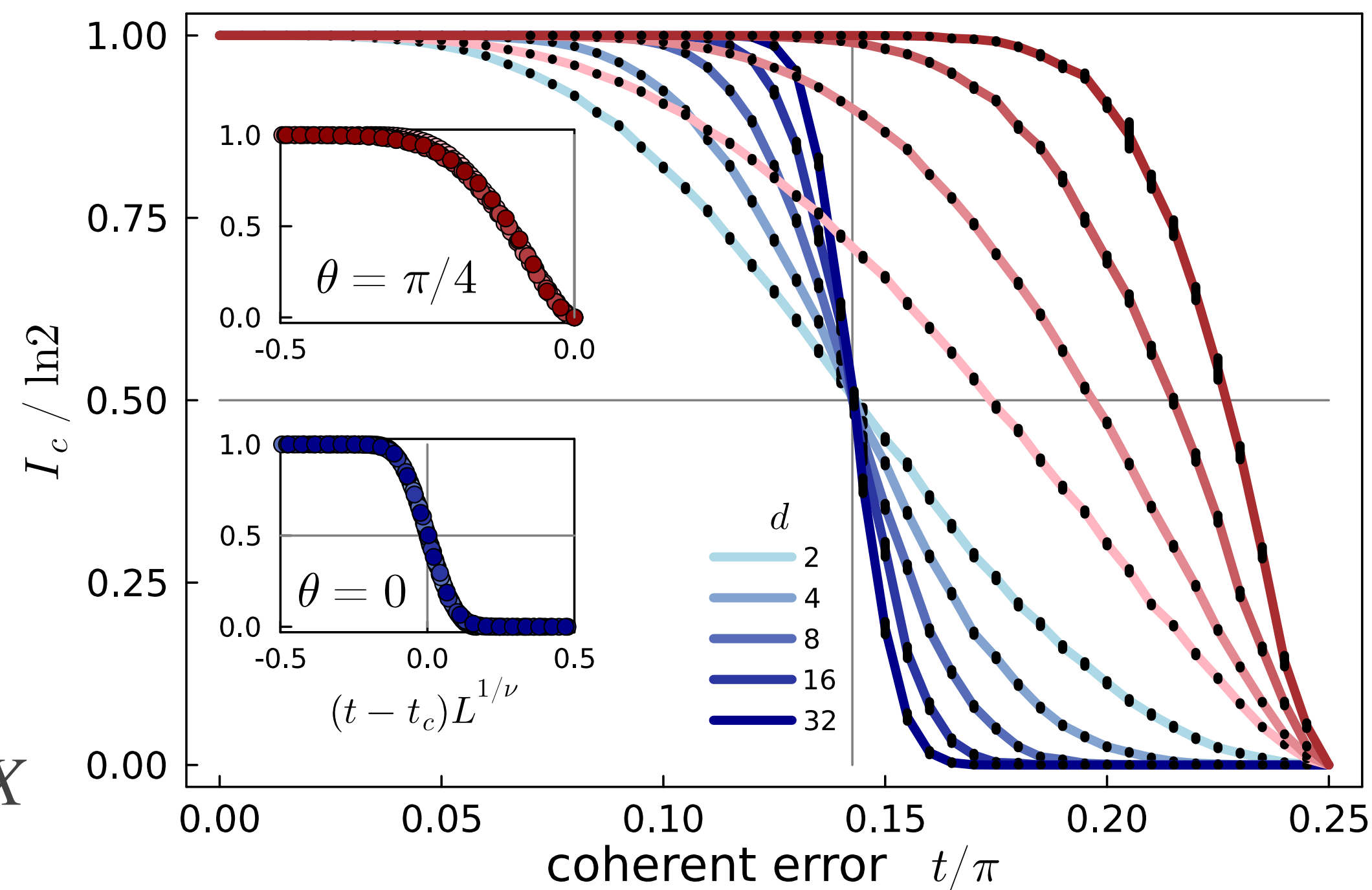
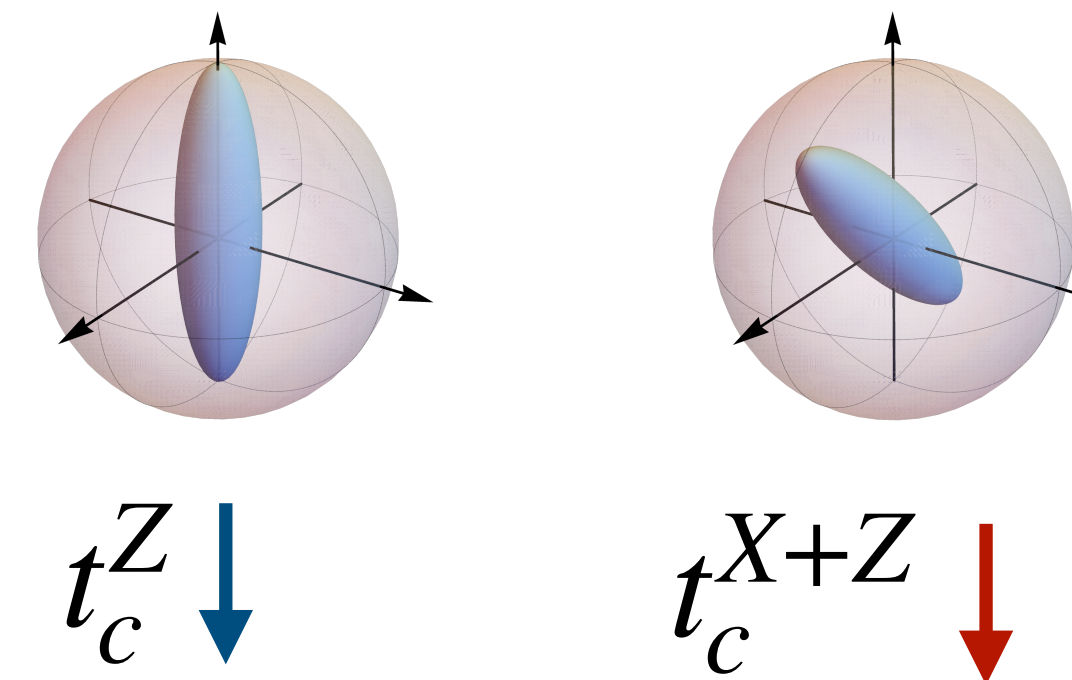
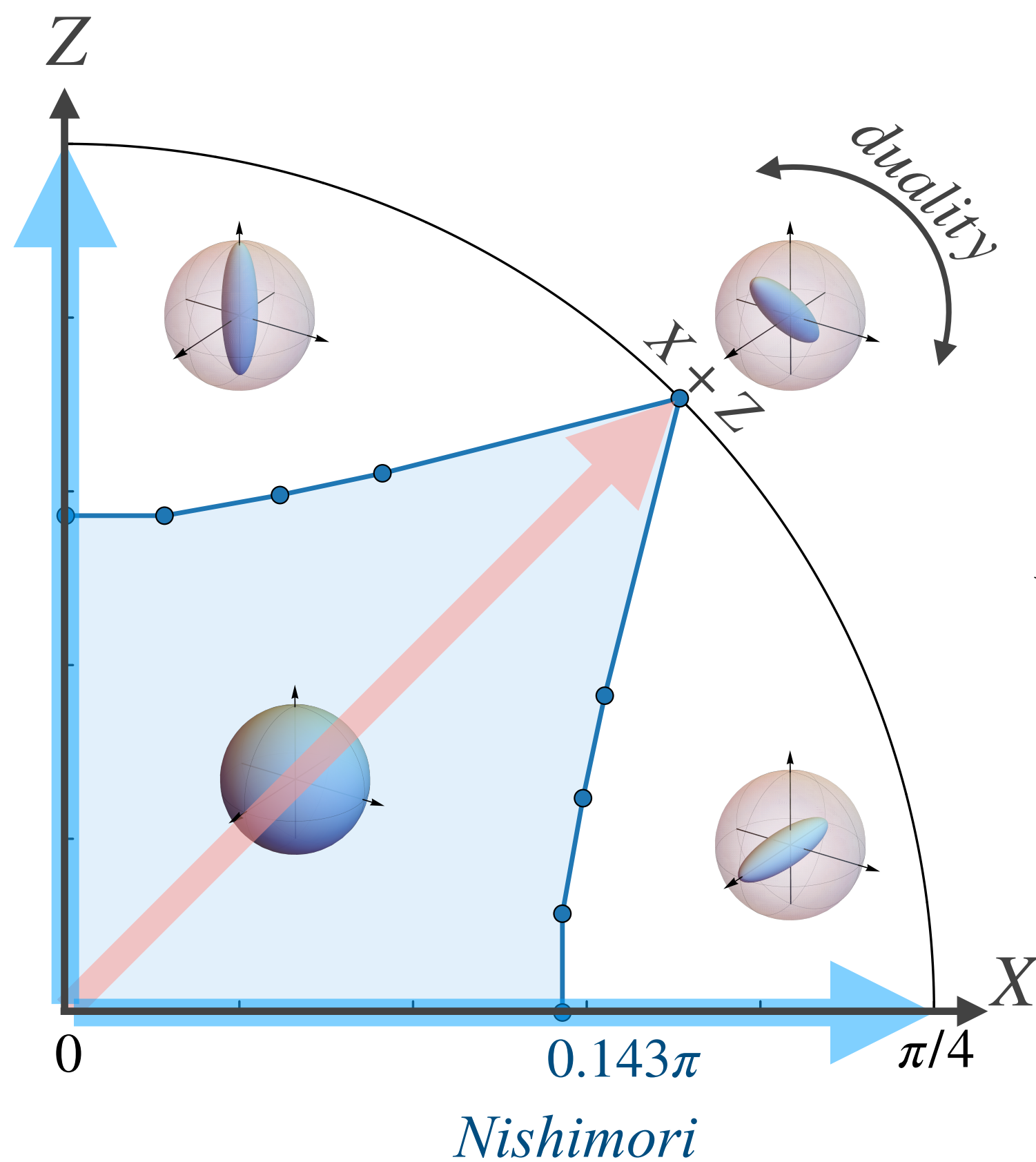
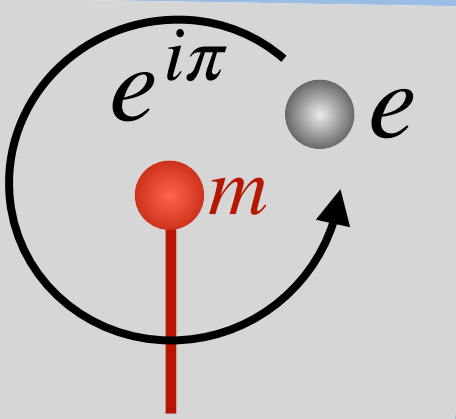
Born's rule $\langle\Psi| M_s^2 |\Psi\rangle$



physical qubits / teleportation vs. measurement



thresholds / phase transitions



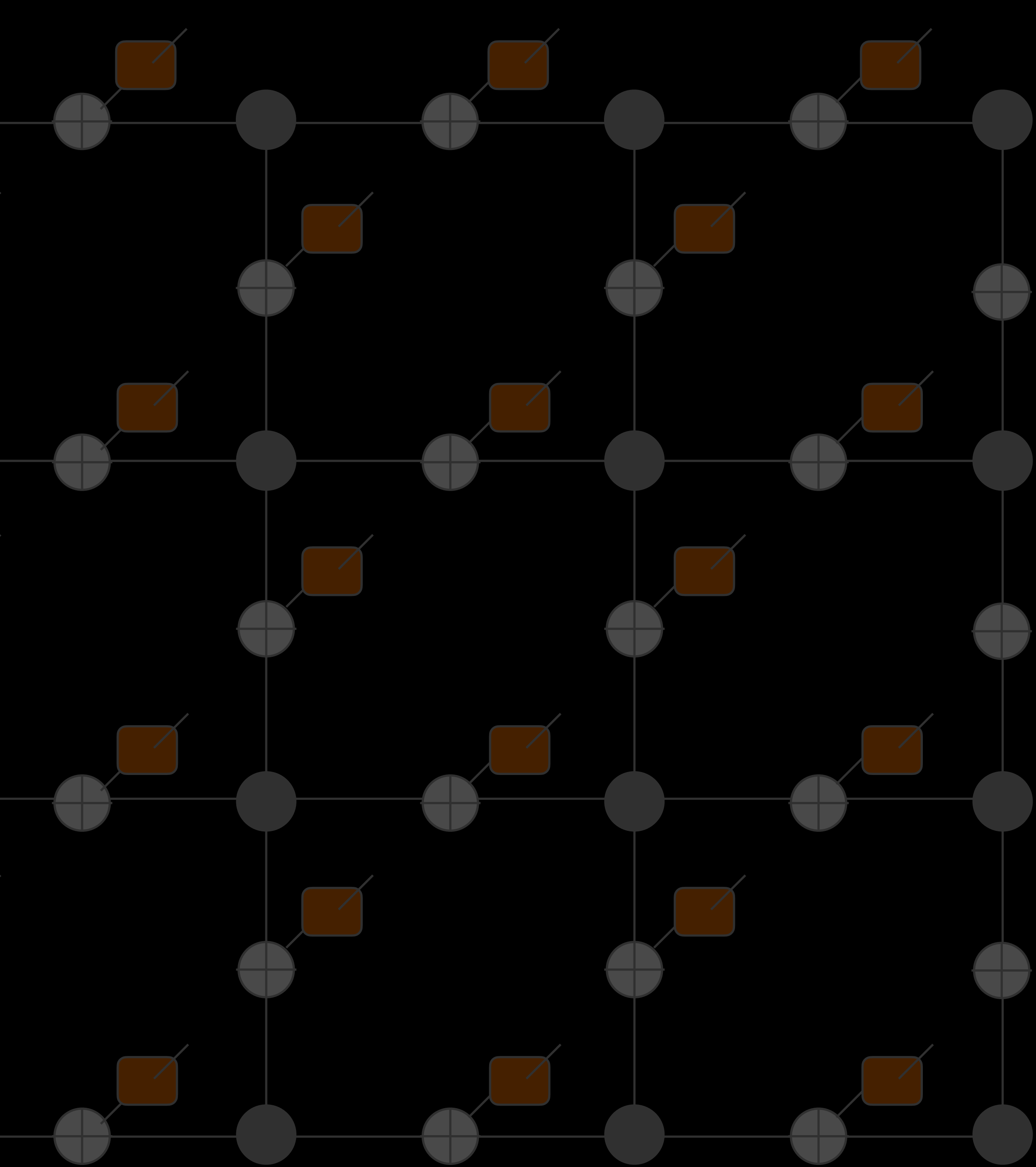
$$\nu = 1.6(1)$$

$$\nu = 1.8(1)$$

angle dependence
due to **competition** of
anyon condensation

self-dual direction
 ∞ threshold

teleportation succeeds even
for **infinitesimal coupling**

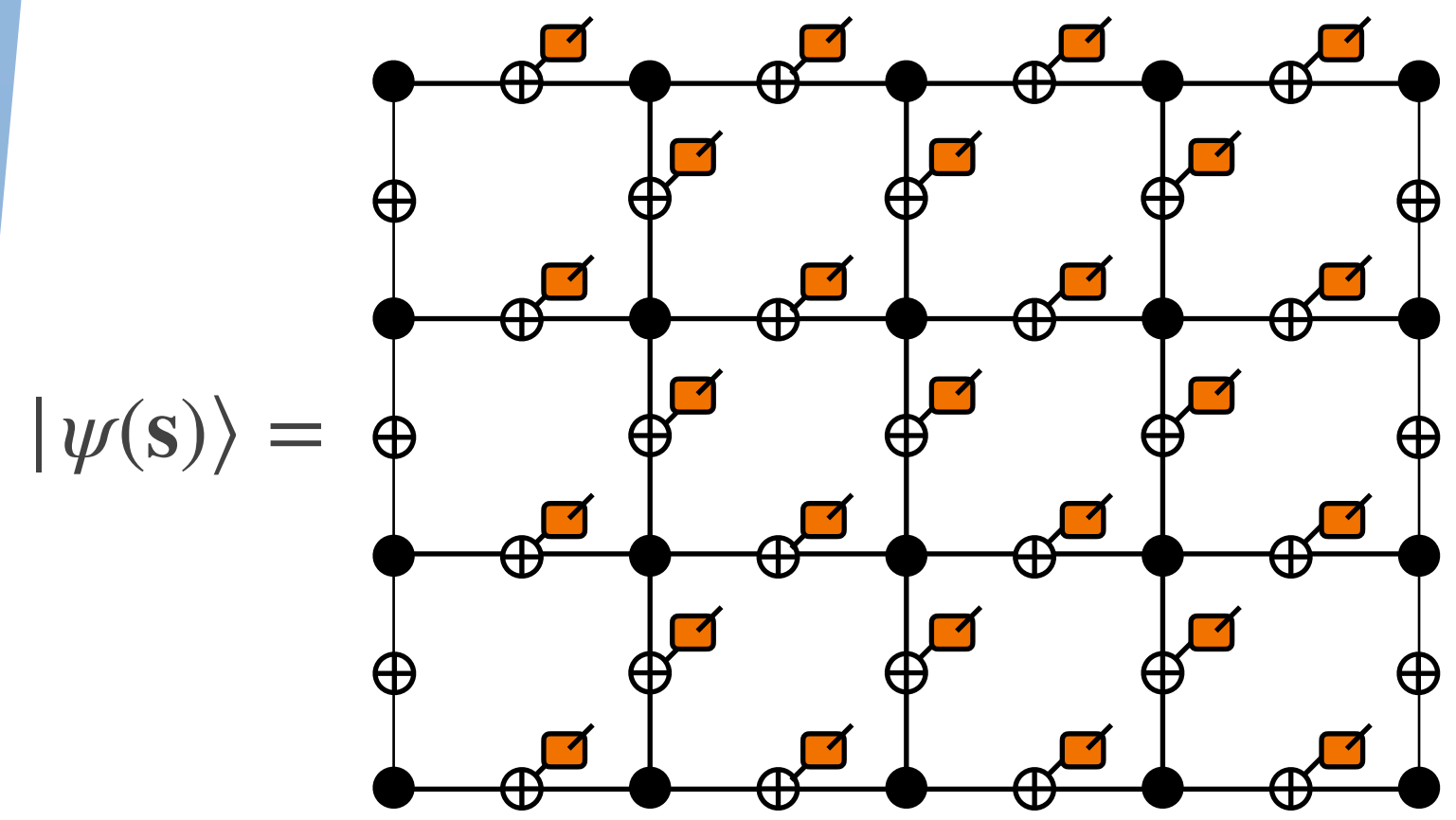


**stat mech
perspective**

tensor network & statistical model

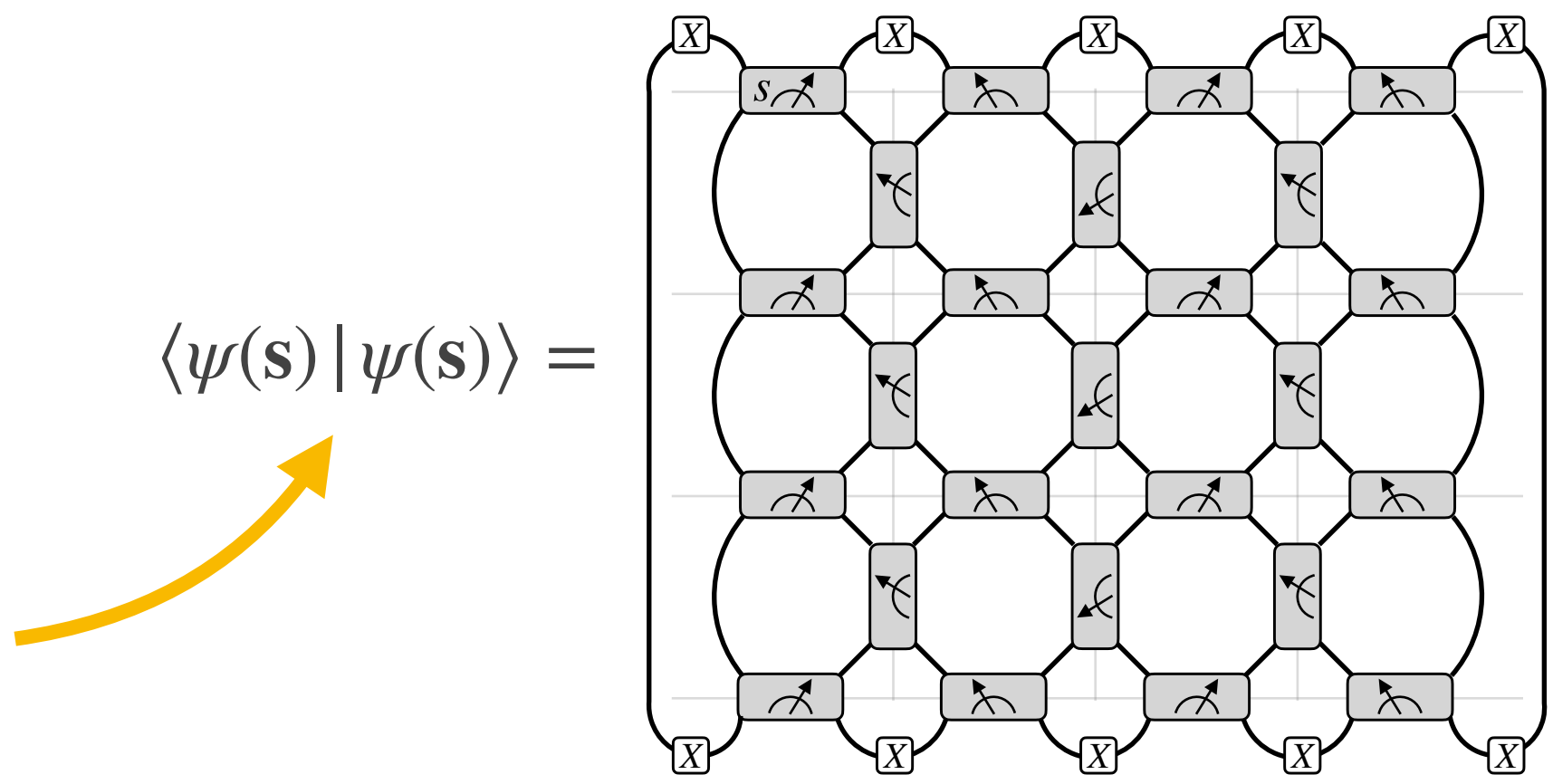
(2+0) dimensional

deformed wave function



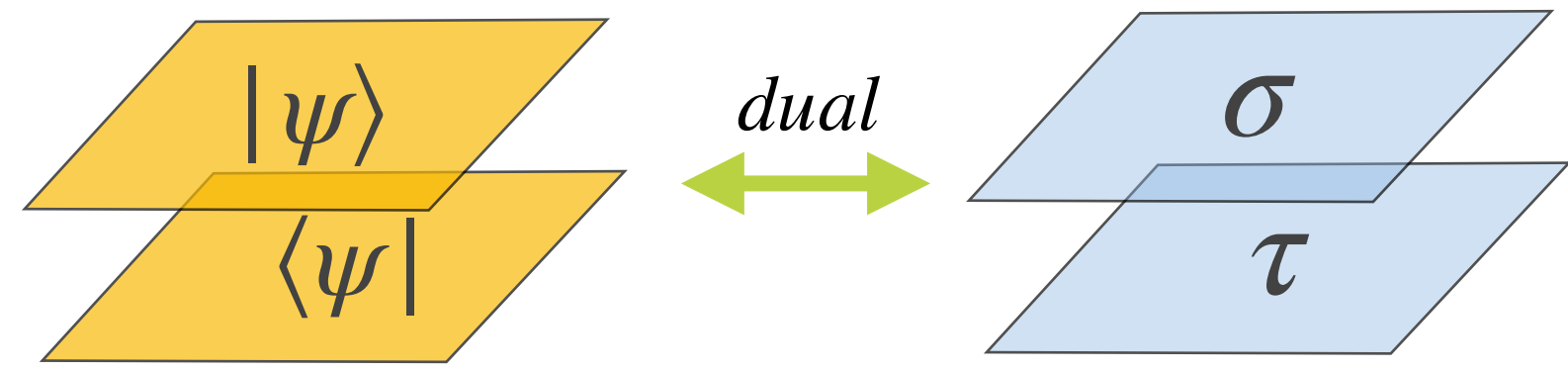
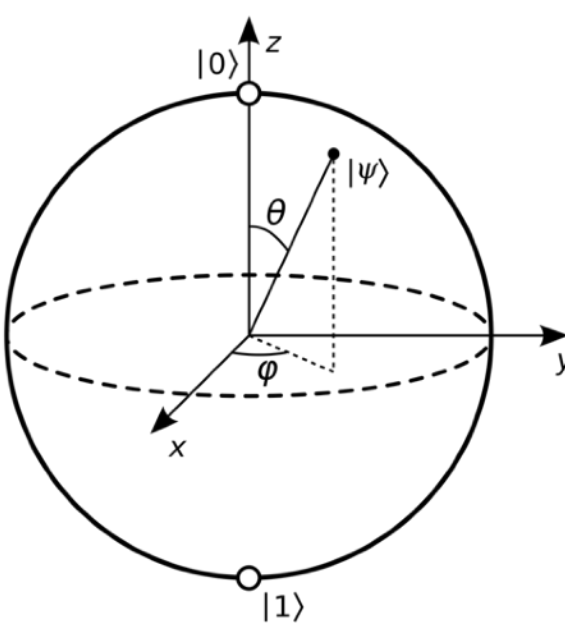
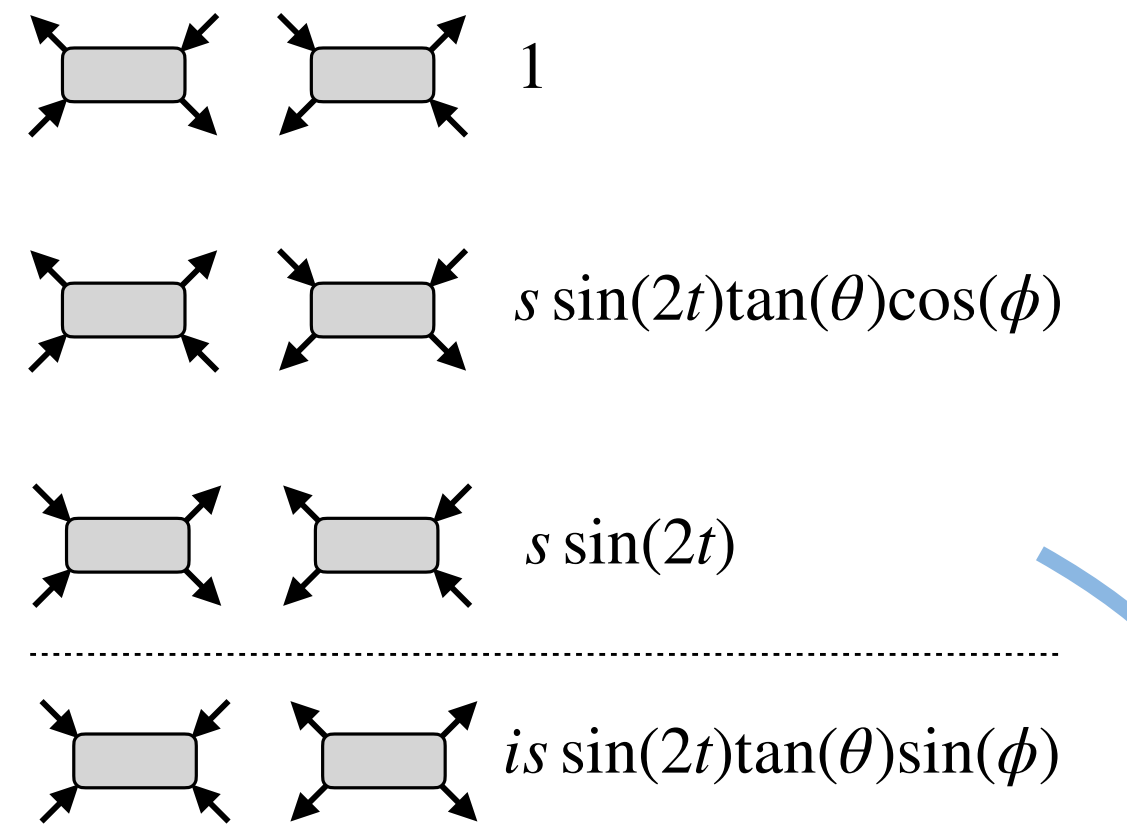
(1+1) dimensional

random circuit



2-dimensional

classical 8-vertex model



$$\tanh J = \sin(2t)\cos(\theta)$$

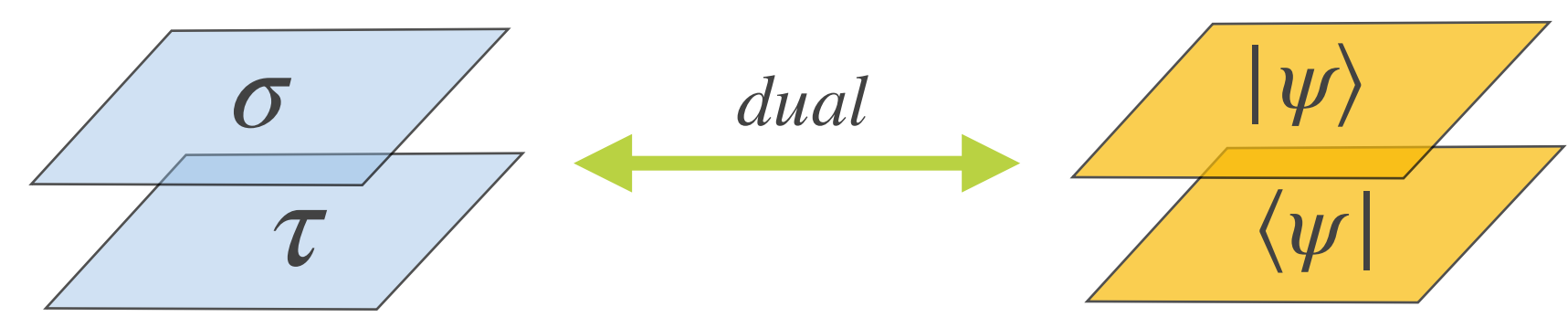
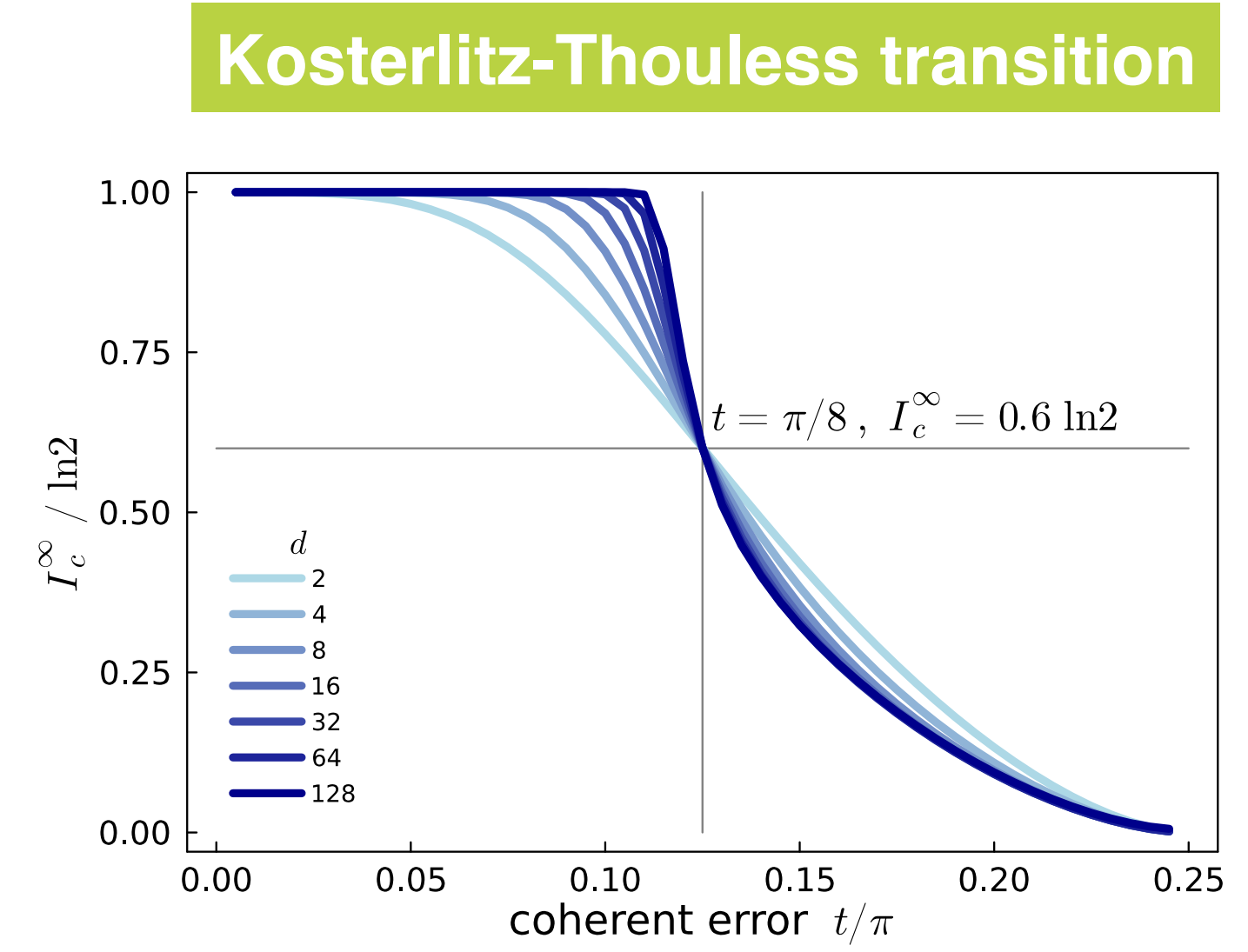
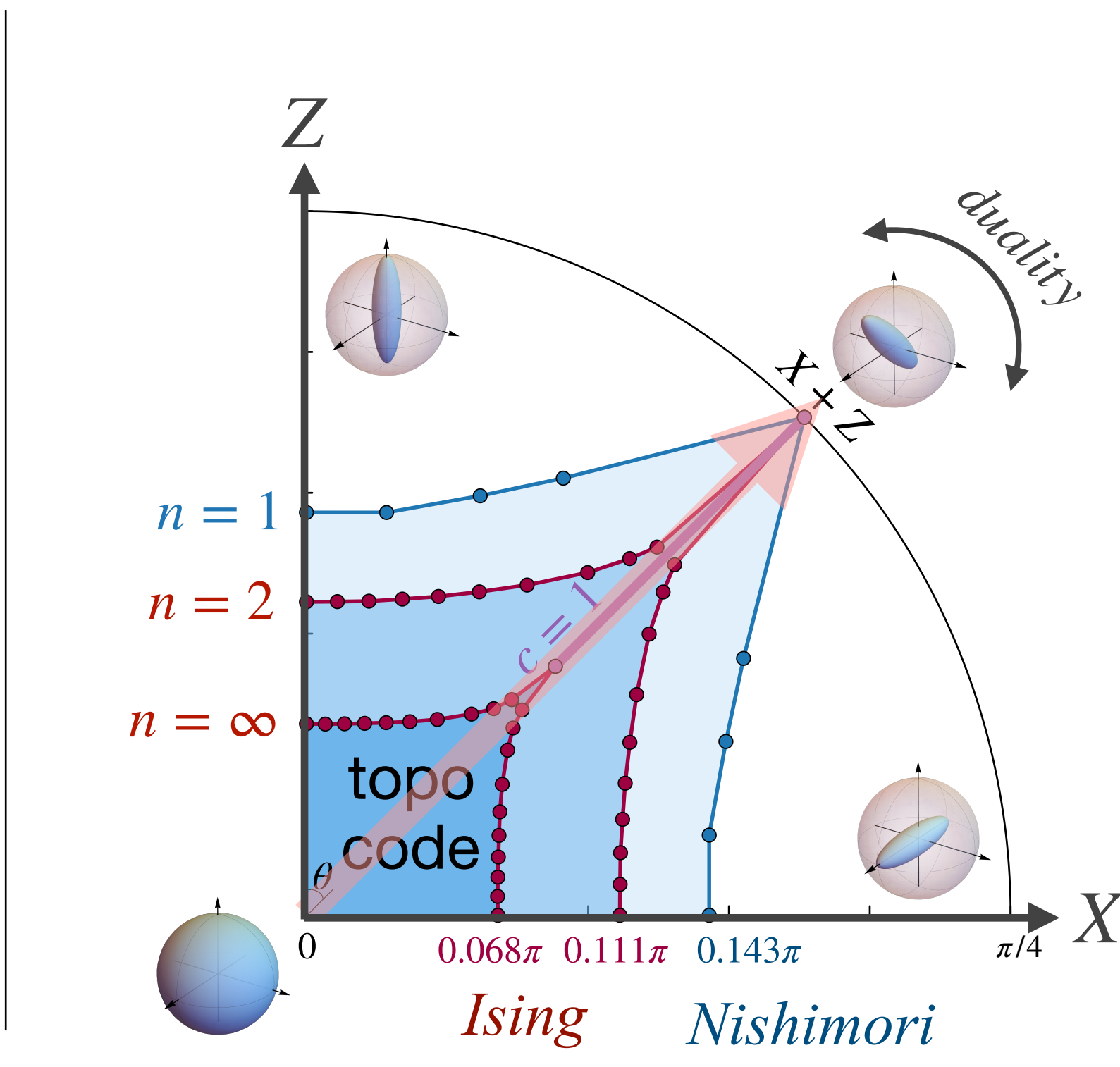
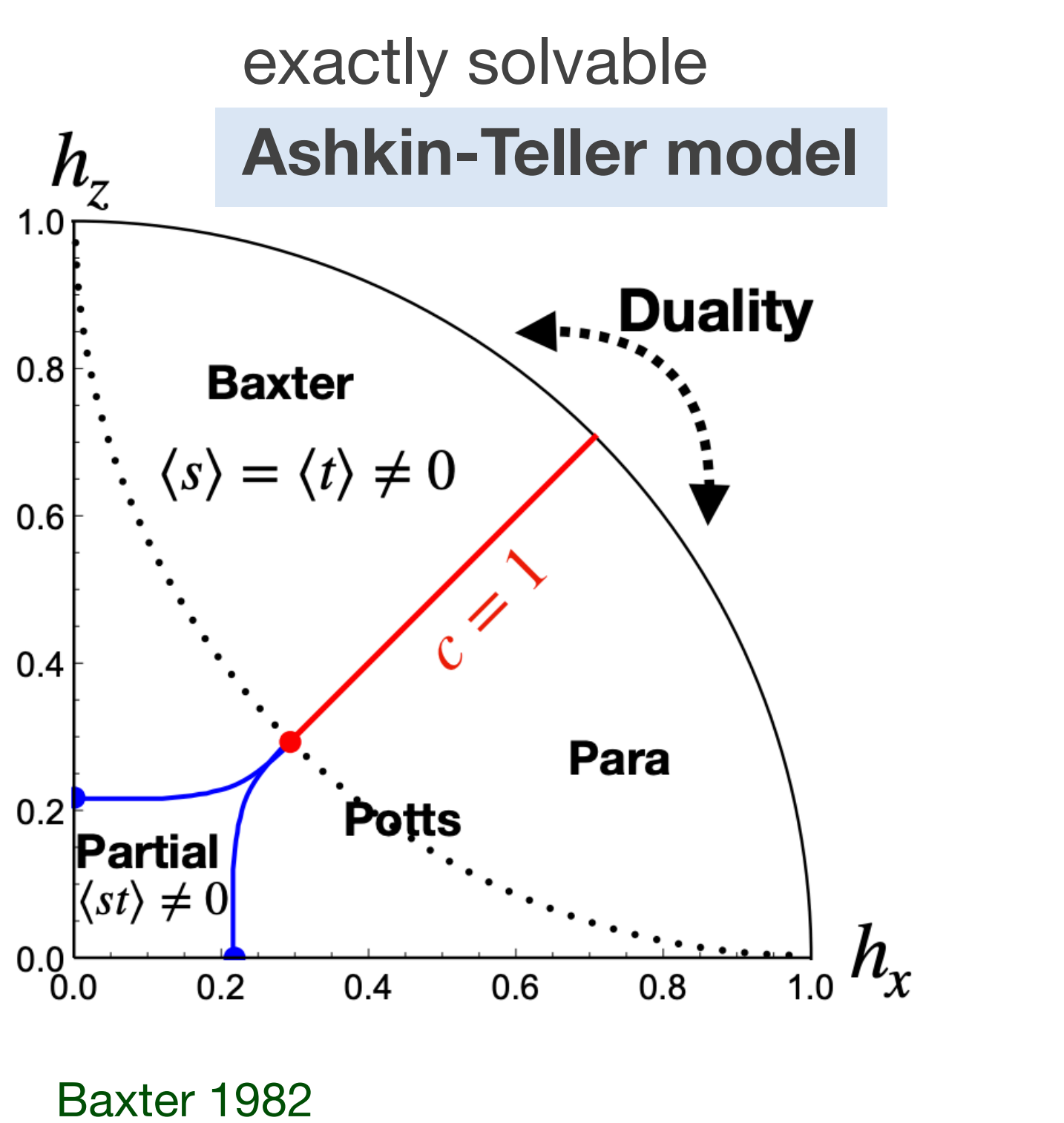
$$e^{-2K} = \sinh(J)\tanh(\theta)$$

$$-E_{ij} = Js_{ij} \frac{\sigma_i \sigma_j + \tau_i \tau_j}{2} + i\phi \frac{\sigma_i \sigma_j - \tau_i \tau_j}{2} + \left(2K + i\pi \frac{1 - s_{ij}}{2} \right) \frac{\sigma_i \sigma_j \tau_i \tau_j - 1}{2}$$

Ashkin-Teller model

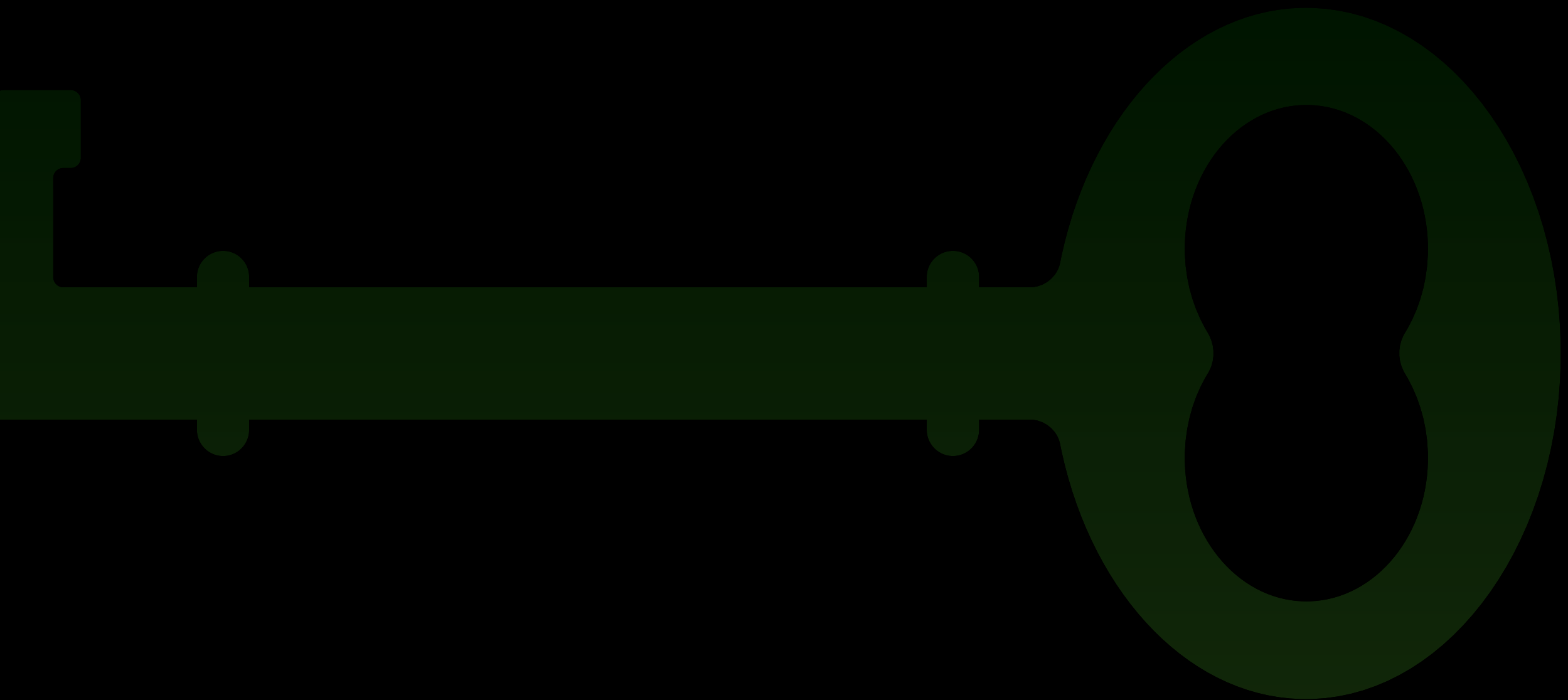
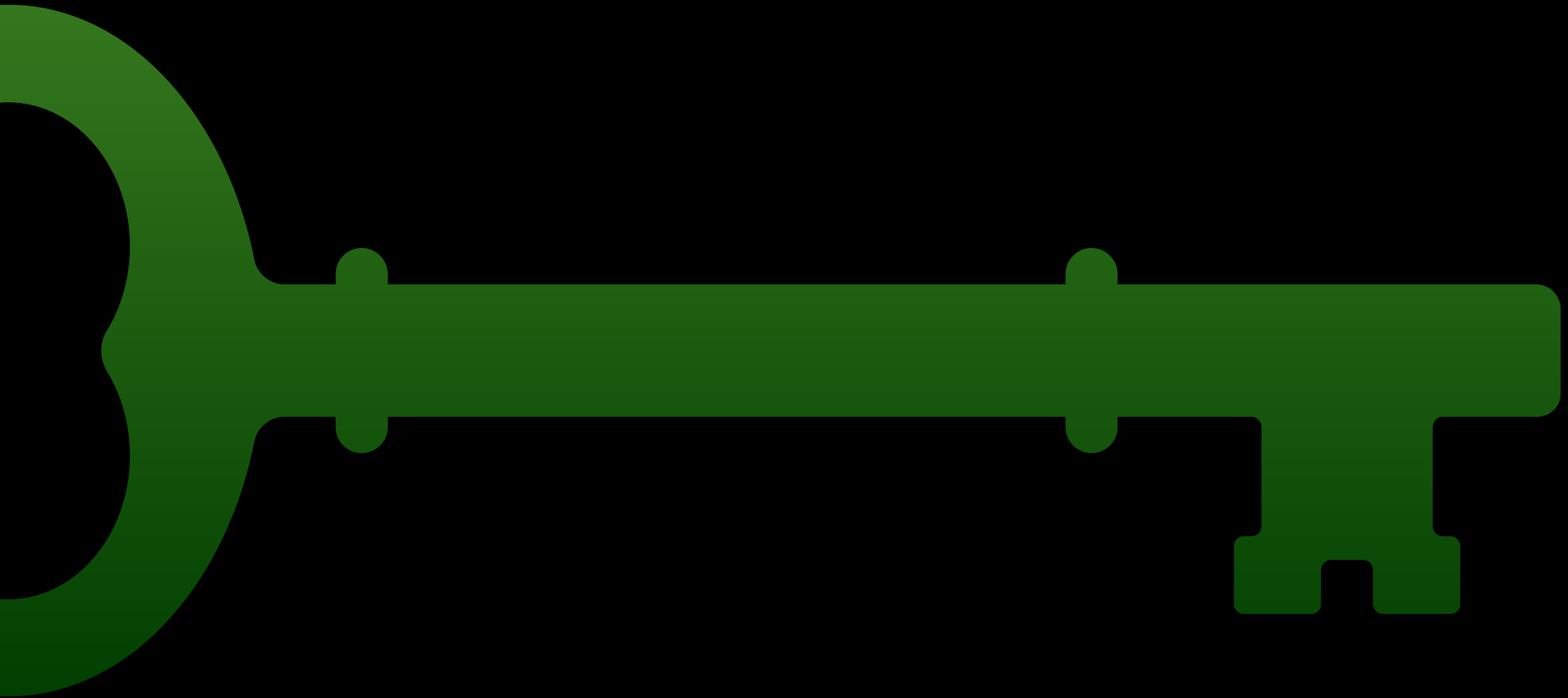
∞ -replica model / post-selection

$\sum_s P(s)^\infty$ distills out **most probable** configuration $s = +1$ \rightarrow **no randomness**



wave function deformation

Ardonne, Fendley, Fradkin 2004; Zhu & Zhang, PRL 2019

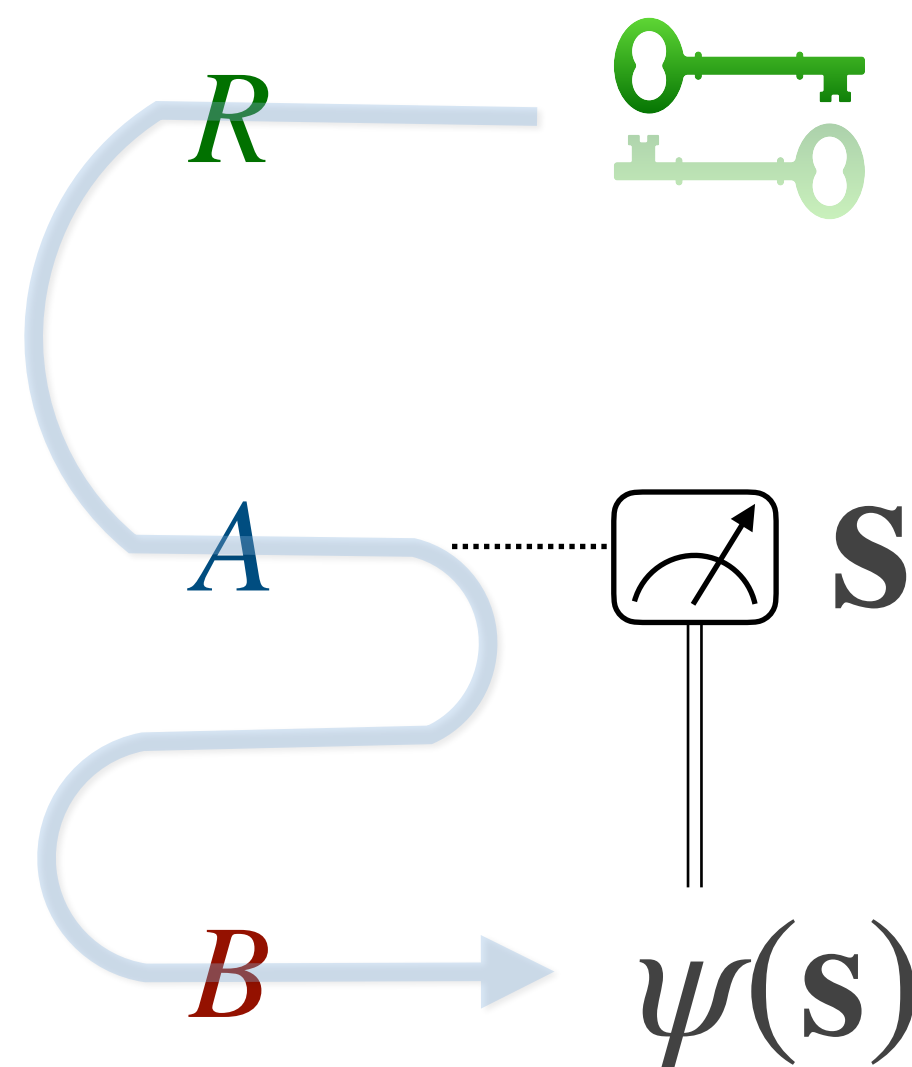


**decoding
teleportation**

decoding many-qubit teleportation

Who has the key / logical qubit?

- A quantum key **cannot be cloned**.
- Does the logical information **leak to Alice** or **flow to Bob** ?



Alice?

$$I_c = S_{RA} - S_A$$

Bob?

$$I_c = S_{AB} - S_{RAB}$$

Scalable decoder
for entanglement transition
Gullans & Huse, PRL 2020

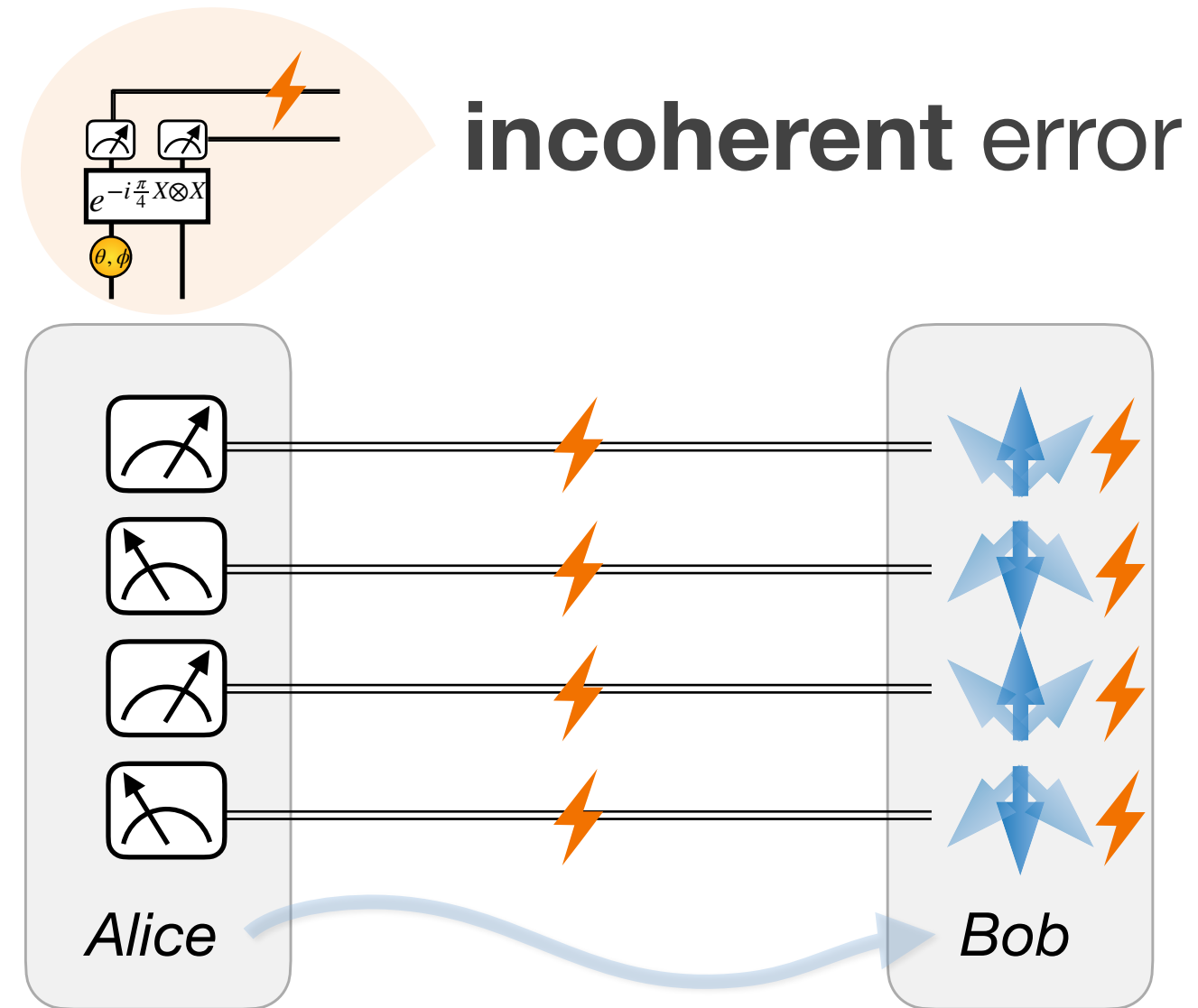
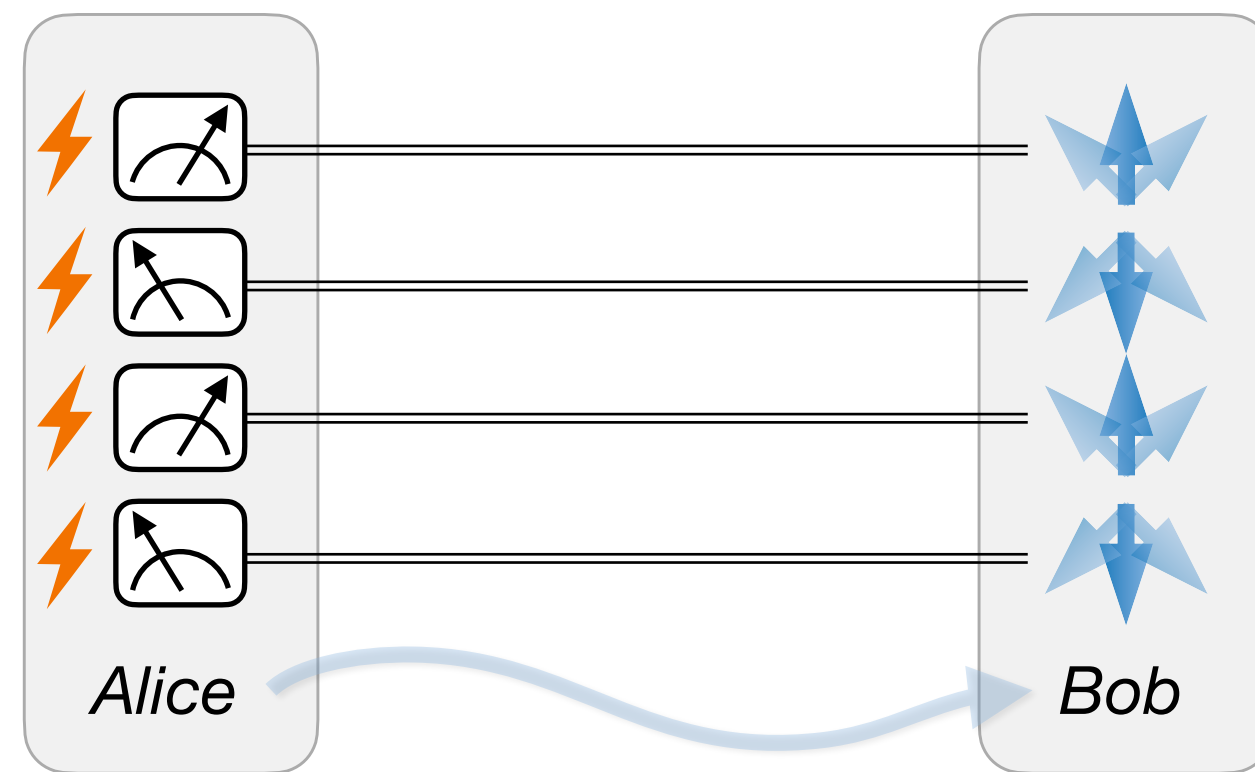
error correction ?

decoding many-qubit teleportation

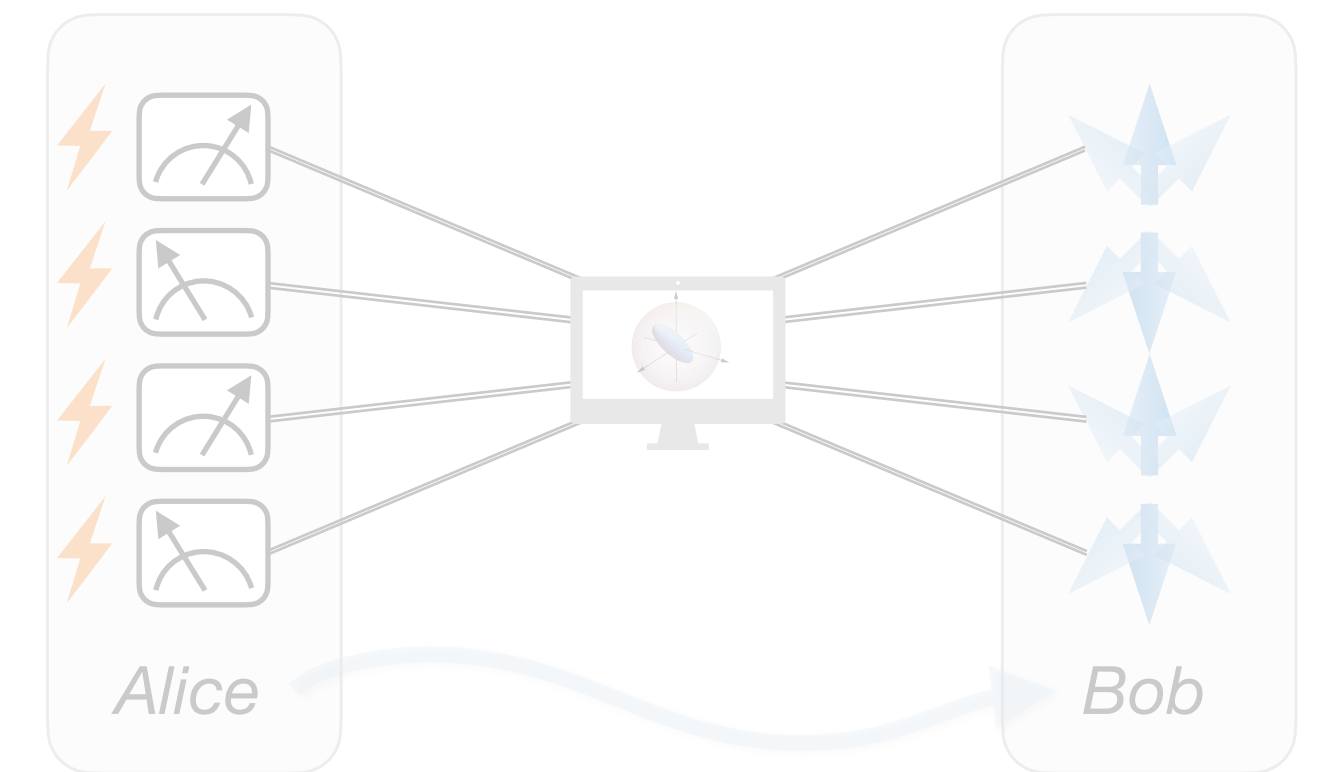
passive teleportation

active teleportation

coherent error w/o decoder



coherent error w/ decoder



Z error
 $\theta = 0$

$$t_c = 0.107\pi$$

$$p = \sin^2(t)$$

$$p_c = 10.9\%$$

$$t_c = 0.143\pi$$

$\theta = \pi/4$
X + Z error

$$t_c = \pi/4$$

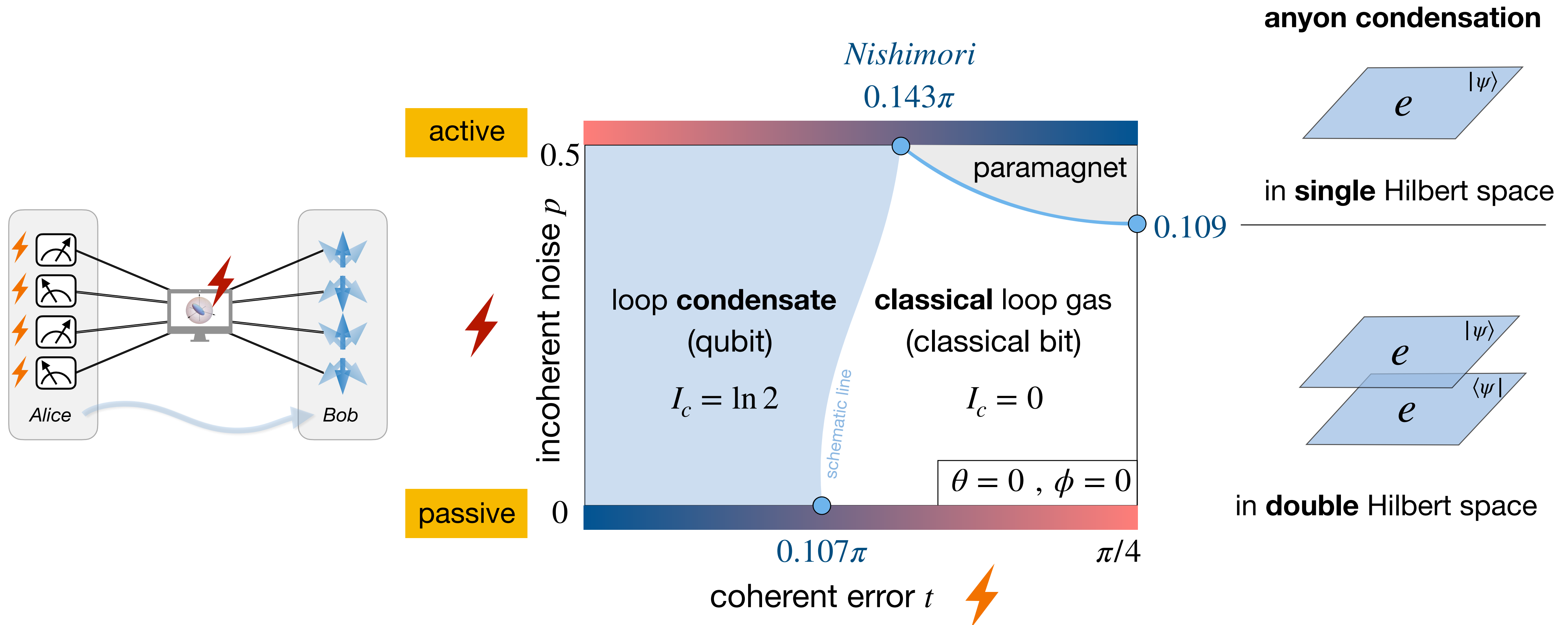
$$p_c = 50\%$$

$$t_c = \pi/4$$

(2-replica result, see also Chen & Grover '24)

(our finding)

interpolating active & passive teleportation

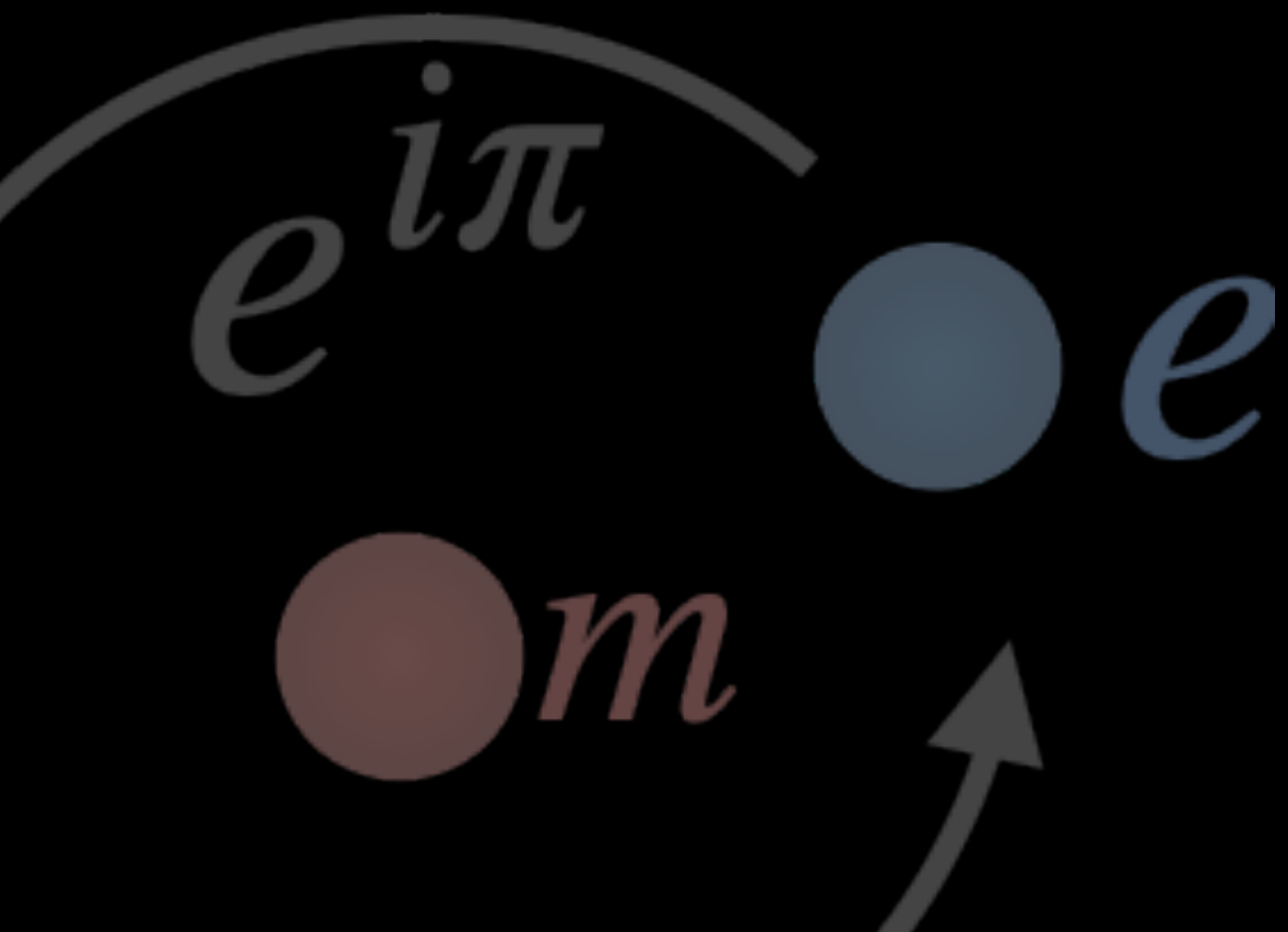


dual to Nishimori's cat realized in **IBM experiment**

Chen, GYZ, Verresen, Seif, Bäumer, Layden, Tantivasadakarn, Zhu, Sheldon, Vishwanath, Trebst, Kandala 2023

Dennis, Kitaev, Landahl, Preskill 2002; Fan, Bao, Vishwanath, Altman 2023; Lee, Jian, Xu, 2023; Li, Mong, 2023; ...

summary



conclusions



Guo-Yi Zhu

- **teleportation of many-body state** / logical qubit

self-duality \Rightarrow **optimal threshold**

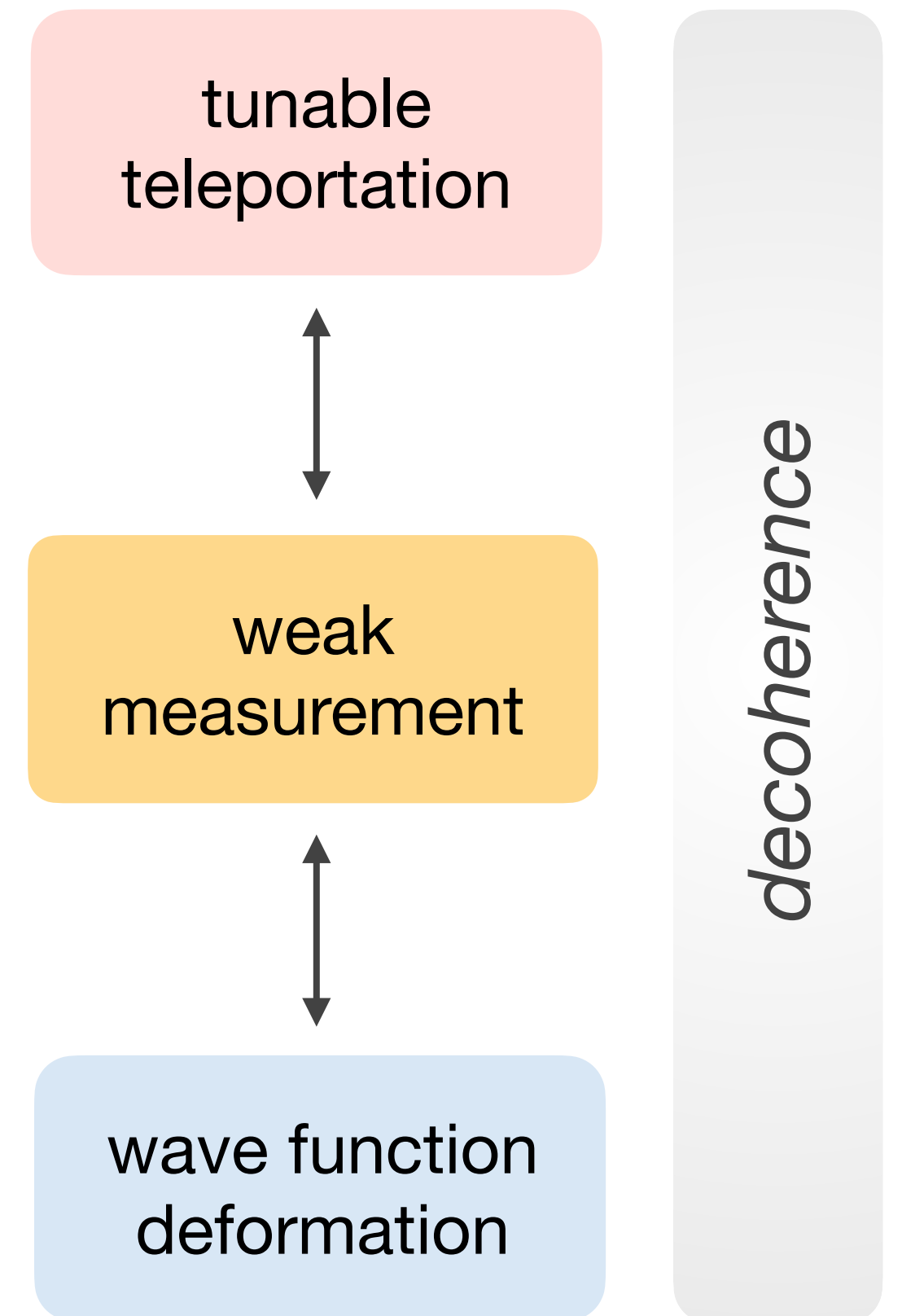
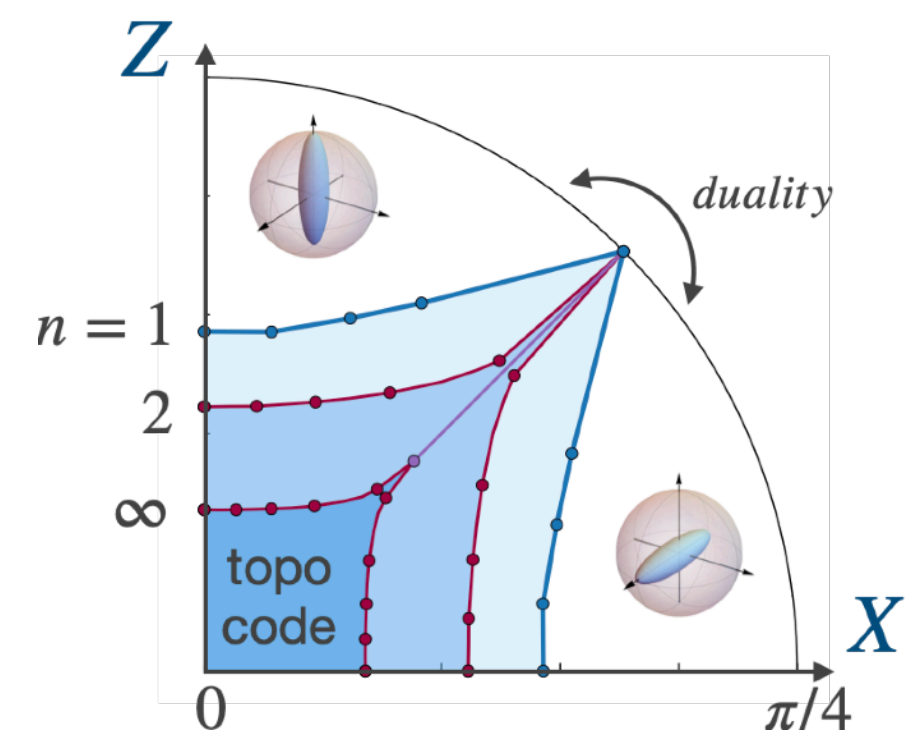
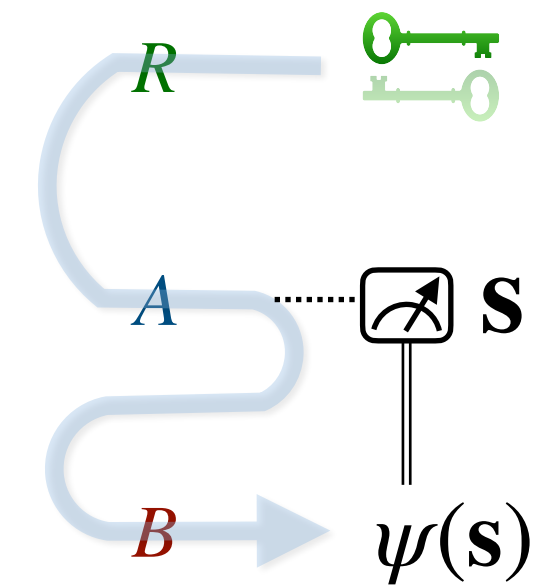
- **topological order**

competing **anyon condensation** phase transitions

- **spin glass model**

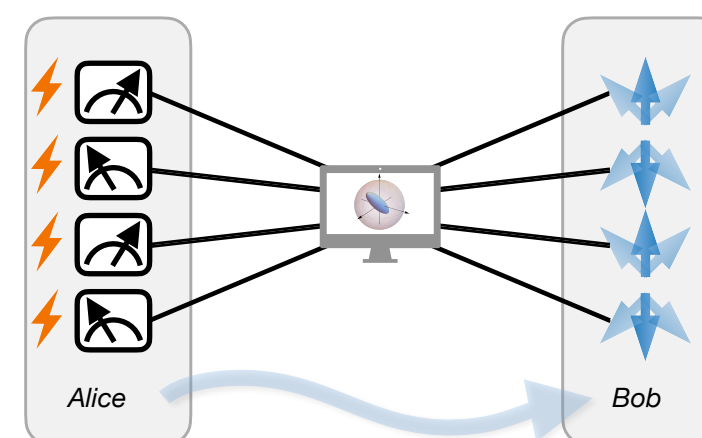
self-dual 4-state Potts | complex Ashkin-Teller model

- **experimentally feasible** in multiple NISQ platforms



Outlook

- non-unitary CFTs & non-Hermitian topology?
- coherent error + incoherent noise?



F. Eckstein, B. Han, ST, G.Y. Zhu
PRX Quantum 5, 040313 (2024)