

Graph neural network based decoders for quantum error correcting codes

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2<sup>nd</sup> Workshop of Machine Learning for Quantum Technology Max Planck Institute for the Science of Light Erlangen Nov 6, 2024





### Quantum computing limited by decoherence

Superconducting qubits:

- lifetime 100 microsecond
- two qubit gate times few 100 nanoseconds
   Maximum few 100 gates deep circuits (error rates ~ 10<sup>-3</sup>)

To factor N= $2^{2048}$  size integer using Shor's algorithm takes >  $(logN)^2 = 10^7$  deep circuit. (error rates <  $10^{-7}$ )





Longer qubit lifetimes needed to get ``quantum advantage''?

Assuming all gates

readily available!

### Quantum error correction

Peter W. Shor, "Scheme for reducing decoherence in quantum computer memory," Physical Review A **52**, R2493–R2496 (1995).

A. M. Steane, "Error Correcting Codes in Quantum Theory," Physical Review Letters **77**, 793–797 (1996). Daniel Gottesman, "Stabilizer Codes and Quantum Error Correction," (1997), arXiv:quant-ph/9705052.

#### Distribute information over many physical qubits --> Lower error rate logical qubit

Article Realizing repeated quantum error correction in a distance-three surface code



9 qubit "surface code"



|0>

### Scalable error suppression

Quantum error correction below the surface code threshold

Google Quantum AI and Collaborators (Dated: August 27, 2024)



49 qubit surface code

Best results rely on machine learning for decoding! Bausch et al. 2023

### Outline

- The surface code and the decoding problem
- Matching Decoders
- Why machine learning decoders?
- Graph neural networks
- Results and work in progress



### Surface code recap

Planar version of Kitaev's toric code

- $n=d^2$  (data) qubits
- $d^2$ -1, 4 and 2 qubit stabilizer (generators)
- Commuting and independent
- *k*=1 logical qubit
- Code-distance d



- Hilbert space partitioned by the ±1 eigenvalues of the stabilizers into d<sup>2</sup>-1, 2dimensional sectors
- Any of these can serve as the logical qubit

S. B. Bravyi and A. Y. Kitaev, Quantum codes on a lattice with boundary (1998), arXiv:quant-ph/9811052.
E. Dennis, A. Kitaev, A. Landahl, and J. Preskill, Topological quantum memory, Journal of Mathematical Physics 43, 4452 (2002).

A. Kitaev, Fault-tolerant quantum computation by anyons, Annals of Physics **303**, 2 (2003).

 $Z_{L}|0\rangle_{L} = |0\rangle_{L}$   $|1\rangle_{L} = X_{L}|0\rangle_{L}$   $\rho = \sum_{i,j \in \{0,1\}} \rho_{ij}|i\rangle_{L}\langle j|_{L}$ 

Logical Pauli operators:

commute with stabilizers

outside stabilizer group

minimal undetectable error=code-distance

### **Decoding basics**



#### Decoder: Syndrome => Correction

Challenge: 2<sup>d^2+1</sup> errors (Pauli strings) consistent with any 1 syndrome!

### Equivalence classes of errors

X,



- Errors within a class are equivalent, since the logical qubit is an eigenstate of any stablizer
- Optimal decoder: suggest a correction from the most likely class

### Estimating class probabilities



Probability of an error chain C, with  $n_c$  errors:  $\pi_C = (p/3)^n (1-p)^{N-n} = (1-p)^N (\frac{p/3}{1-p})^n = (1-p)^N e^{-n/T} \qquad 1/T = -\ln(\frac{p/3}{1-p})^n$ 

Probability of an equivalence class *E*:

 $P_E \sim Z_E = \sum_{C \in E} e^{-n_C/T}$ 

Optimal, Maximum-likelihood decoder: calculate the partition functions

### Asymptotic logical failure rate

Assume error rate  $p \ll 1$ 

Most likely error

Most likely error in other class







When this error occurs => logical (bit-flip) failure

Asymptotic logical failure:  $P_f \sim p^{(d+1)/2}$ 

Logical errors are exponentially supressed with code distance d

### Decode<sup>---</sup>, MLD versus MLE

#### MLD: Maximum-likelihood decoder.

#### Metropolis - Monte Carlo

PRL 109, 160503 (2012)

week ending 19 OCTOBER 2012

#### High Threshold Error Correction for the Surface Code

PHYSICAL REVIEW LETTERS

James R. Wootton and Daniel Loss Department of Physics, University of Basel, Klingelbergstrasse 82, CH-4056 Basel, Switzerland (Received 1 March 2012; published 18 October 2012)

#### PHYSICAL REVIEW A 105, 042616 (2022)

#### Error-rate-agnostic decoding of topological stabilizer codes

Karl Hammar<sup>®</sup>,<sup>1</sup> Alexei Orekhov<sup>®</sup>,<sup>2</sup> Patrik Wallin Hybelius<sup>®</sup>,<sup>1</sup> Anna Katariina Wisakanto,<sup>1</sup> Basudha Srivastava<sup>®</sup>,<sup>1</sup> Anton Frisk Kockum<sup>®</sup>,<sup>2</sup> and Mats Granath<sup>®</sup>,<sup>1,\*</sup> <sup>1</sup>Department of Physics, University of Gothenburg, 41296 Gothenburg, Sweden
<sup>2</sup>Department of Microtechnology and Nanoscience, Chalmers University of Technology, 1296 Gothenburg, Sweden

#### **Tensor network based**

PHYSICAL REVIEW A **90**, 032326 (2014) **Efficient algorithms for maximum likelihood decoding in the surface code** 

> Sergey Bravyi, Martin Suchara, and Alexander Vargo IBM Watson Research Center, Yorktown Heights, New York 10598, USA (Received 23 June 2014; published 25 September 2014)

Accurate but slow.



MLE: Most likely error decoder

#### Matching decoders (next slide)



D. S. Wang, A. G. Fowler, A. M. Stephens, and L. C. L. Hollenberg, Threshold error rates for the toric and planar codes, Quantum Inf. Comput. **10**, 456 (2010).

D. S. Wang, A. G. Fowler, and L. C. L. Hollenberg, Surface code quantum computing with error rates over 1%, Phys. Rev. A **83**, 020302(R) (2011).

#### Suboptimal, but fast

### Matching decoders (Dijkstra + Blossom)



PyMatching: A Python package for decoding quantum codes with minimum-weight perfect matching

Oscar Higgott<sup>\*1</sup>

### **Belief-matching**

Accounts for correlations between X and Z stablizers, due to Y errors



#### PHYSICAL REVIEW X 13, 031007 (2023)

Improved Decoding of Circuit Noise and Fragile Boundaries of Tailored Surface Codes

Oscar Higgott<sup>(0)</sup>,<sup>1,2,\*</sup> Thomas C. Bohdanowicz,<sup>3,4</sup> Aleksander Kubica,<sup>4,5</sup> Steven T. Flammia,<sup>4,5</sup> and Earl T. Campbell<sup>2,6,7</sup>

Improved accuracy Slower

### Stabilizer measurement circuits

To measure stabilizers we use ancilla (measure) qubits



#### XXXX stabilizer





### Simulating experiments with circuit-level noise

We use Stim to generate simulated "experiments"





#### Stim: a fast stabilizer circuit simulator

#### Craig Gidney

oogle Inc., Santa Barbara, California 93117, USA				
Published:	2021-07-06, <b>volume 5</b> , page 497			
Eprint:	arXiv:2103.02202v3			
Doi:	https://doi.org/10.22331/q-2021-07-06-497			
Citation:	Quantum 5, 497 (2021).			



### Logical failure rates for matching

At overall error rate  $p=1.0 \times 10^{-3}$ 



Very high accuracies!

Requires detailed knowledge of error channel

# Q. Can we use a data driven machine learning approach to decoding?

Motivated by e.g. natural language processing where large deep learning models made strutured (gramatical) approaches obsolete.

- Model free! (non-Pauli error channel)
- Potentially fast and scalable

Extreme requirements on:

- Accuracy 0.999999 (or even higher)
- Inference time µs

### Previous work on deep learning based decoders

TABLE I. A comprehensive literature survey and the comparison of the machine learning based syndrome decoders.

Paper	Error correction	dmax	Threshold	ML Technique	Noise model
	code	-mux			
A scalable and fast artificial neural	Surface code with	1025	0.138	Supervised learning con-	Depolarizing, Inho-
network syndrome decoder for surface	boundaries, braid-			volution neural network	mogeneous and Bi-
codes [This Work]	ing and lattice				ased noise models
	surgery structures				
Scalable Neural Decoder for Topologi-	Toric code	255	0.162(5)	Supervised learning dense	Depolarizing noise
cal Surface Codes [22]				neural network	1 0
Reinforcement learning for optimal er-	Toric code	9	0.103	Reinforcement learning,	Bit-flip
ror correction of toric codes [45]				Deep convolutional net	
Neural Network Decoders for Large-	Toric code	64	0.095	Supervised learning,	Bit flip
Distance 2D Toric Codes [35]				Renormalization group	
				based neural network	
Neural ensemble decoding for topologi-	Surface code	11	Not	Supervised learning,	Depolarizing noise
cal quantum error-correcting codes [46]			reported	Neural network ensemble	
				learning	
Deep Q-learning decoder for depolariz-	Toric code	9	0.165	Deep reinforcement	Depolarizing noise
ing noise on the toric code [47]				learning	
Comparing neural network based de-	Rotated surface	9	0.146	Supervised learning, Feed	Depolarizing and
coders for the surface code [48]	code		(depol.),	forward neural networks,	circuit noise
			0.0032(circ.)	Recurrent neural nets	
				with LSTMs	
Symmetries for a High Level Neural De-	Toric code	7	Not	Supervised learning, Feed	Depolarizing noise
coder on the Toric Code [49]			reported	forward neural net	
Quantum error correction for the toric	Toric code	7	Not	Deep reinforcement	Bit-flip
code using deep reinforcement learning			reported	learning	
[50]					
Decoding surface code with a dis-	Rotated surface	9	Not	Supervised learning, Neu-	Depolarizing noise
tributed neural network based decoder	code		reported	ral network	
[51]					
Reinforcement Learning Decoders for	Rotated surface	5	Not	Reinforcement learning,	Bit-flip, Depolar-
Fault-Tolerant Quantum Computation	code		reported	Convolutional neural	izing, Phenomeno-
[52]				network	logical noise
Deep neural decoders for near term	Rotated surface	5	Not	Supervised learning, Deep	Circuit noise
fault-tolerant experiments [24]	code		reported	neural networks, Single	
				layer neural networks	
Scalable Neural Network Decoders for	3D toric code, 4D	12	0.175 (3D),	Supervised learning, Con-	Bit-flip, Phe-
Higher Dimensional Quantum Codes	toric code		0.071 (4D)	volutional neural network	nomenological
[53]					noise
Machine-learning-assisted correction of	Rotated surface	3	Not	Supervised learning, Re-	Depolarizing noise
correlated qubit errors in a topological	code		reported	current neural net with	and Measurement
code [21]				LSTMs	errors
Decoding small surface codes with feed-	Rotated surface	7	Not	Supervised learning, Feed	Bit-flip, Depolar-
forward neural networks [17]	code		reported	forward neural network	izing, Phenomeno-
					logical and Circuit
					noise
Deep Neural Network Probabilistic De-	Toric code	9	0.164	Neural net with 15-18 hid-	Depolarizing noise
coder for Stabilizer Codes [20]				den layers	
Neural Decoder for Topological Codes	Toric code	6	0.109	Restricted Boltzmann	Phase-flip errors
1110	1	1	1	machine	1

#### Neural Decoder for Topological Codes

Giacomo Torlai and Roger G. Melko Phys. Rev. Lett. **119**, 030501 – Published 18 July 2017

#### A scalable and fast artificial neural network syndrome decoder for surface codes

Spiro Gicev,<sup>1, \*</sup> Lloyd C.L. Hollenberg,<sup>1, †</sup> and Muhammad Usman<sup>1, 2, ‡</sup>

 <sup>1</sup>Center for Quantum Computation and Communication Technology, School of Physics, University of Melbourne, Parkville, 3010, VIC, Australia.
 <sup>2</sup>School of Computing and Information Systems, Melbourne School of Engineering, University of Melbourne, Parkville, 3010, VIC, Australia

#### Mostly conceptual, with simplified error models

### Our early attempt: Deep Reinforcement Learning



Quantum error correction for the toric code using deep reinforcement learning

Philip Andreasson, Joel Johansson, Simon Liljestrand, and Mats Granath

Accepted in { }uantum 2019-08-24, click title to verify

Reinforcement learning decoders for fault-tolerant quantum computation

Ryan Sweke<sup>1</sup> <sup>[</sup><sup>®</sup>], Markus S Kesselring<sup>1</sup>, Evert P L van Nieuwenburg<sup>2</sup> <sup>[</sup><sup>®</sup>] and Jens Eisert<sup>1,3</sup> Published 28 December 2020 · © 2020 The Author(s). Published by IOP Publishing Ltd <u>Machine Learning: Science and Technology, Volume 2, Number 2</u> Step-by-step correction



PHYSICAL REVIEW RESEARCH 2, 023230 (2020)

#### Deep Q-learning decoder for depolarizing noise on the toric code

David Fitzek © <sup>1,2,4</sup> Mattias Eliasson.<sup>3</sup> Anton Frisk Kockum © <sup>1</sup> and Mats Granath ®<sup>3,1</sup> <sup>1</sup>Wallenberg Centre for yountum Technology, Department of Microtechnology and Nanoscience, Chalmers University of Technology, SE-41296 Gothenburg, Sweden <sup>2</sup>Yolvo Group Tracks Technology, 405 08 Gothenburg, Sweden <sup>3</sup>Department of Physics, University of Gothenburg, Se-1296 Gothenburg, Sweden

- Inefficient
- Difficult to scale
- Misses the point

### Recent work: Graph neural network decoder



Tailored to experimental input

Fast inference? => in time error correction High Accuracy => high logical fidelity



Data-driven decoding of quantum error correcting codes using graph neural networks

Moritz Lange,<sup>1</sup> Pontus Havström,<sup>1</sup> Basudha Srivastava,<sup>1</sup> Valdemar Bergentall,<sup>1</sup> + Isak Bengtsson Karl Hammar,<sup>1</sup> Olivia Heuts,<sup>1</sup> Evert van Nieuwenburg,<sup>2,\*</sup> and Mats Granath<sup>1,†</sup>

<sup>1</sup>Department of Physics, University of Gothenburg, SE-41296 Gothenburg, Sweden <sup>2</sup>Leiden Inst. of Advanced Computer Science, Leiden University, Leiden, Netherlands

#### arXiv:2307.01241

### Memory-Z experiment

- 1. Simple product state prepared
- Stabilizers are measured over several rounds => changes=detector events Perfect Z-stabilizers in first and final rounds
- 3. Final individual qubit measurement
- 4. Measured logical coset of error (given by parity change on designated edge) compared to decoder prediction
   Gives logical fidelity of the quantum memory

#### **GNN** decoder data

- Detector events = graph nodes
  - space-time location and type as node feature vector
- Edges ~ inverse euclidean (or manhattan) distance
- Label: binary class, logical bit-flip or not (or logical phase-flip or not)

Pruning of edges based on edge weights





### Graph neural networks (GNN)

Neural networks suited for graph structured data

Article

#### Examples

Antibiotic discovery graph regression

Authors

In Brief

Jonathan M. Stokes, Kevin Yang, Kyle Swanson, ..., Tommi S. Jaakkola,

Regina Barzilay, James J. Collins

A trained deep neural network predicts

antibiotic activity in molecules that are structurally different from known antibiotics, among which Halicin exhibits

efficacy against broad-spectrum bacterial infections in mice.

Correspondence

jimjc@mit.edu (J.J.C.)

regina@csail.mit.edu (R.B.),

#### Cell

#### A Deep Learning Approach to Antibiotic Discovery

Graphical Abstract



#### Highlights

- A deep learning model is trained to predict antibiotics based on structure
- Halicin is predicted as an antibacterial molecule from the Drug Repurposing Hub
- Halicin shows broad-spectrum antibiotic activities in mice
- More antibiotics with distinct structures are predicted from the ZINC15 database

### Cora dataset, citation network Node classification



McCallum et al. 2000

### Data object: Decorated graph

Node feature vectors



### Graph convolutional layers

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Grid: Standard convolutional filter of fixed size neighborhood



Simple graph convolution:

$$\mathbf{x}'_i = \sigma(W_1 \mathbf{x}_i + W_2 \sum_j e_{ij} \mathbf{x}_j)$$

 $W_1$  and  $W_2$ : n'x n trainable weight matrices

Semi-supervised classification with graph convolutional networks Kipf and Wellling, 2016 Graph: Convolutional filter adapted to varying neighborhood

A comprehensive survey on graph neural networks Wu et al. 2019  $\mathbf{X'}_i$ ٠  $\boldsymbol{X}_i$ ٠ ٠ ٠ • ٠ ٠ • ٠ ٠ ٠ . ٠ ٠ . ٠

### Graph pooling

For graph classification output should be independent of number of nodes

$$\mathbf{x}' = \frac{1}{\# nodes} \sum_i \mathbf{x}_i$$



### GNN decoder network architecture

5



**PyG** (*PyTorch Geometric*) is a library built upon **O PyTorch** to easily write and train Graph Neural Networks (GNNs) for a wide range of applications related to structured data.

### GNN on circuit-level noise

Lacking sufficient experimental data we use simulated "experiments"







### Training

- One network for each code distance d and number of cycles  $d_t$
- Range of training error rates p=1.0-5.0 x 10<sup>-3</sup>
- Data generated in large batches of 10,000-25,000 graphs (as much as can fit on the GPU memory)
- No reuse of data (no risk to overfit)
- Up to one week training on one Nvidia A100
- Up to 10<sup>10</sup> datapoints



Test

Benchmarked against matching (MWPM) and belief-matching at p=1.0 x 10<sup>-3</sup>



- GNN probably close to optimal (maximum-likelihood) decoder for small d
- Matching decoders know the error model, the GNN decoder does not
- Scaling to larger d is challenging, work in progress using larger networks

### GNN as a maximum-likelihood decoder

work in progress



### Test optimality

preliminary results

- 1. Bin GNN data according to predicted class probability
- 2. Compare to actual failure rates



Predicted Logical Class Probabilities VS Average Logical Failure Rate at Different Distances

GNN estimates class probabilities accurately. Indication that it's close to optimal.

### Most likely failures

#### Failure count versus decoder confidence



Interestingly, syndromes with high and low decoder confidence all contribute significantly to logical errors

# Potential use-case: GNN as soft-output decoder for concatenated codes

Of interest to output not only most likely class, but also the probability of failure

#### Yoked surface codes

Craig Gidney<sup>1</sup>, Michael Newman<sup>1</sup>, Peter Brooks<sup>2</sup>, and Cody Jones<sup>1</sup>

Hierarchical memories: Simulating quantum LDPC codes with local gates

Christopher A. Pattison<sup>1</sup>, Anirudh Krishna<sup>2,3</sup>, and John Preskill<sup>1,4</sup>



- Surface code concatenated with other low-density parity check (LDPC) code
- Outer code decoder (with matching or belief propagation) can use conditional inner code error propabilities

### Decoding low-density parity check (LDPC) code

work in progress

### High-threshold and low-overhead fault-tolerant quantum memory

778 | Nature | Vol 627 | 28 March 2024



- Encodes 12 logical qubits in 144 physical qubits
- Non-local stabilizers in 2D
- Non-matchable (hyperedges)

### GNN decoder for LDPC codes

preliminary results

- Multiple logical qubits => multiple output layer nodes
- All graph nodes (stabilizers) are proximate due to long-range connectivity



Still a little way to go!

### Inference time per syndrome



- Fixed size network
- Hardware and implementation dependent

- Decoding time scales linearly with code "volume"
- NB Both MWPM (Pymatching) and GNN are batched/parallelized

GNN on real experimental data



Repetition code, 25 stabilizer cycles Datsets of around 10<sup>7</sup>



GNN decoder on par with "informed" matching decoder for d=3 and 5.



Surface code dataset is too small, and/or error rates too high.

### Other recent related work

#### Neural network decoder for near-term surface-code experiments

Boris M. Varbanov,<sup>1, \*</sup> Marc Serra-Peralta,<sup>1, 2</sup> David Byfield,<sup>3</sup> and Barbara M. Terhal<sup>1, 2</sup> <sup>1</sup>QuTech, Delft University of Technology, P.O. Box 5046, 2600 GA Delft, The Netherlands <sup>'</sup>Delft Institute of Applied Mathematics, Technische Universiteit Delft, 2628 CD Delft, The Netherlands <sup>3</sup>Riverlane, Cambridge, CB2 3BZ, United Kingdom (Dated: October 24, 2023)



#### Learning to Decode the Surface Code with a Recurrent, Transformer-Based Neural Network

Johannes Bausch<sup>1\*†</sup>, Andrew W Senior<sup>1\*†</sup>, Francisco J H Heras<sup>1†</sup>, Thomas Edlich<sup>1†</sup>, Alex Davies<sup>1†</sup>, Michael Newman<sup>2†</sup>, Cody Jones<sup>2</sup>, Kevin Satzinger<sup>2</sup>, Murphy Yuezhen Niu<sup>2</sup>, Sam Blackwell<sup>1</sup>, George Holland<sup>1</sup>, Dvir Kafri<sup>2</sup>, Juan Atalaya<sup>2</sup>, Craig Gidney<sup>2</sup>, Demis Hassabis<sup>1</sup>, Sergio Boixo<sup>2</sup>, Hartmut Neven<sup>2</sup>, Pushmeet Kohli<sup>1</sup>

<sup>1</sup>Google DeepMind & <sup>2</sup>Google Quantum AI



## 9 Oct 2023

### Work in progress: "Neural belief-matching"

- A pure neural network decoder is very data-hungry
- Can we combine a smaller graph network with a matching decoder? Still data-driven/model-free

#### GNN provides edge-weights to a matching decoder

![](_page_38_Figure_4.jpeg)

 Challenge: Loss is nondifferentiable as matching gives discrete output

![](_page_38_Figure_6.jpeg)

Approaching MWPM with error informed edge-weights

### Conclusions

- Data-driven model-free approach to decoding using graph neural network
- Competitive to matching decoders for accuracy and speed
- Approaches maximum-likelihood decoder, soft-output decoder
- Challenging to scale to larger code-distances, more data/larger networks?
- Outlook: Generate training data (using IBM hardware)
- Outlook: Decode sliding window in time to scale (Google is doing this)
- Outlook: Move from GPU to FPGA for fast inference
- Outlook: Hybrid "neural matching decoder"

Collaborators: **Basudha Srivastava** (GU and Quantinuum), **Moritz Lange**, **Isak Bengtsson**, **Blaž Pridgar**, Frida Fjelddahl, Pontus Havström, Valdemar Bergentall, Karl Hammar, Olivia Heuts, David Fitzek (Volvo), Ben Criger (Quantinuum), Anton Frisk Kockum (Chalmers), Evert van Nieuwenburg (Leiden)

![](_page_39_Picture_11.jpeg)

![](_page_39_Picture_12.jpeg)